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Influence of the Setting on the Result of Measuring the Roundness of the Cylindrical and Conical Surface

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When measuring roundness, accurately adjusting the measured part is very important. The axis of the measured part must be perpendicular to the section where the roundness is measured. If this fails, a systematic error will occur. It depends on the size of the inclination of the surface during the measurement. The paper presents mathematical relations to calculate the error when measuring roundness on a cylindrical and conical surface. It analyses the influence of the inclination and the parameters of the measured area on this error. The theoretically determined values are compared with the practically measured roundness values. The error harms the accuracy of the roundness measurement. It affects the value of roundness, but also the roundness profile itself. It is explained that the mistake can not only increase but also decrease the measured value of roundness, in conclusion.

Keywords: Roundness, Systematic Error, Roundness Profile, Cylindrical Surface, Conical Surface

1 Introduction

Measurement accuracy is defined as the closeness of agreement between a measured quantity value and a real quantity value of a measure (i.e., the quantity intended to be measured) and is often limited by calibration errors. The authors [1] discuss the role of precision in process control and product quality. According to them, precision is deemed the most important aspect of process control measurement. Accuracy, if ignored, will have a significant impact on the manufacturer's ability to control the process. The metalworking industry has always put great emphasis on having tools that can increase measurement accuracy. They help with quality control, support reliability, and can support the objective of making it right the first time

However, as manufacturers attempt to grow their trade in the global business environment, the need for measurement accuracy has become an even higher priority due to the time and cost savings that businesses can potentially achieve. Cylindrical components are an indispensable part of engineering products. Roundness is an important fundamental form tolerance for cylindrical components and has a direct influence on the performance of products [2, 3]. However, the tolerance for roundness caused by imperfect manufacturing is not predictable and differs from part to part. Therefore, the precision and reliability of roundness measurement is particularly important, especially in the case of precision and ultra-precision measurements. [4]

The authors in [5] described in detail the mathematical description of the new method based on the small extrusion screw model (SDS) for analytical methods LSC, MIC, MCC, and MZC for roundness evaluation. The experiment was carried out using the roundness of conventional measuring equipment, and the developed SDS method resulted in the evaluation of errors in the form. The results were compared to those obtained using Chebyshev's best-matched reference algorithm.

In [6], the measurement of the comparative evaluation of the roundness profiles and the roundness deviation values of the bearing journals of the main crankshaft of a marine medium speed engine were performed using a correlation calculus. Their proposed system can be used for measurements of form deviations, as well as axis position of cylindrical surfaces of straight shafts and crankshafts.

By [7], to determine the size and geometry errors of high-precision compounds, it is preferable to use non-contact measuring instruments. It is because mechanical measuring instruments lead to deformations, which reduces the accuracy of the results. That is why optical, optoelectronic, laser, and other devices are widely used [8], [9], [10], [11].

The research is focused on the analysis of measurement signals to increase the accuracy of circularity measurement. The article [12] deals with the research of the measuring probe signal of roundness machine Roundscan. It also [13] deals with the determining of the characteristic signal of the inductive FT3 measuring probe, that is used on the roundness machines. The knowledge can be used in the calibration of roundness machines.

2 Defining the problem

When measuring roundness, the plane in which the circular profile of the measure lies must be perpendicular to the axis of the measured part. If the assumption is fulfilled, the measured roundness profile corresponds to the actual roundness profile on the measure component. When measuring, they are clamped to the roundness measurement machine magnetically or mechanically - using a chuck or various clamps. Often, during clamping, an undesired inclination of the measured surface occurs (Fig. 1). The measuring axis is not identical to the axis of the part. When positioning the measured component on the table of the roundness measuring machine, this inclination can cause the end face not to be perpendicular to the axis of the measured rotating surface. It can also cause deformation (bending) of the part.

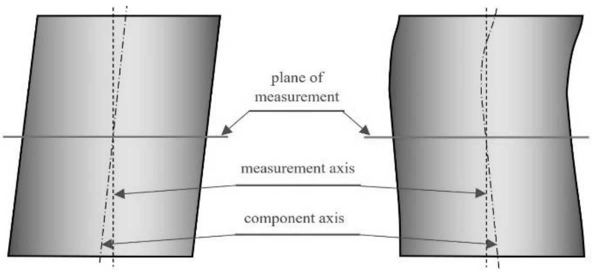

Fig. 1 *The tilt of the measured area*

This inclination of the measured area will affect the measured result of roundness and influence the measured circular profile itself. For example, when measuring the roundness of an ideal cylindrical surface, the roundness profile will have the shape of a circle (if the measured surface is set correctly, the axis of the area is perpendicular to the measuring plane). But when the measured area is tilted, the roundness profile will have the shape of an ellipse (Fig. 2). We can say that an error occurs here, which is mathematically dependent on the size of the inclination of the measured area (the greater the angle of inclination, the larger the error). The error has the character of a systematic error. This is why it is described as a systematic setting error, an error caused by the inaccurate setting of the measured area on the measuring device.

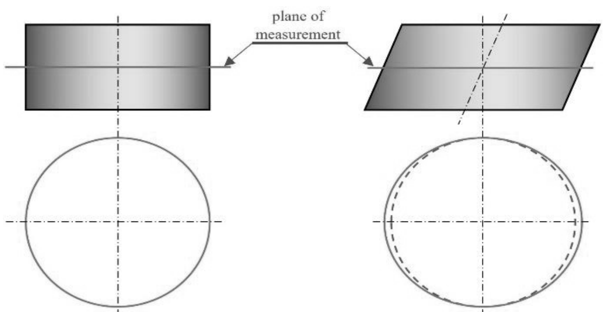

Fig. 2 *Roundness profile when measuring a correctly and incorrectly positioned cylindrical component*

3 Calculation of systematic error of cylindrical surface setting

The systematic positioning error for an ideal cylindrical surface can be expressed mathematically as the difference between the longer and shorter positions of the ellipse, as seen in Fig. 3:

$$\xi = a - b \tag{1}$$

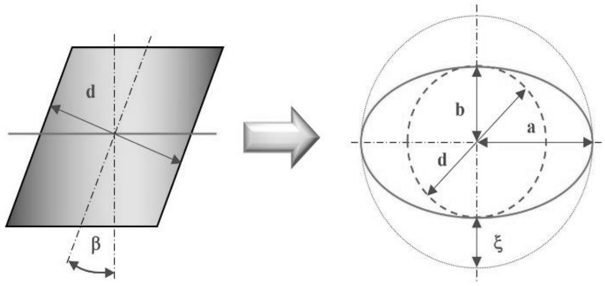

Fig. 3 *Roundness Scheme to determine the systematic error of the cylindrical surface*

Where:
d…Diameter of the measured cylindrical surface,
β…Angle of inclination of the cylindrical surface,
a…Longer half of the ellipse (roundness profile),
b…Shorter half of the ellipse (roundness profile),
ξ…Systematic setting error.
The value of the longer half is:

$$a = \frac{d}{2\cos\beta} \tag{2}$$

The value of the shorter half-axis is equal to the radius of the measured cylindrical surface:

$$b = \frac{d}{2} \tag{3}$$

After substituting relations (2) and (3) into relation (1) we get:

$$\xi = \frac{d}{2}\left(\frac{1}{\cos\beta} - 1\right) \tag{4}$$

Equation (4) applies to the calculation of the systematic setting error for both the outer and inner cylindrical surfaces. This error depends on the diameter *d* and the angle of inclination β of the measured cylindrical surface. Graphically, this dependence is shown in Fig. 4. The systematic positioning error ξ increases with increasing angle β as well as with increasing diameter *d*.

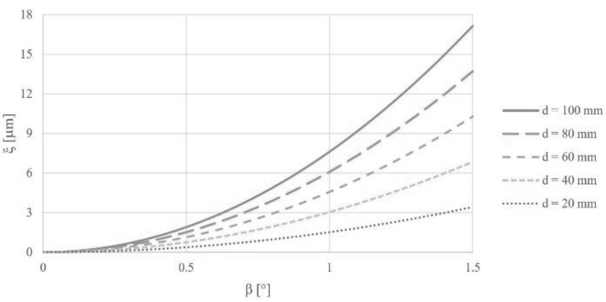

Fig. 4 *Error size ξ for different inclinations β and cylindrical surface diameters d*

4 Calculation of systematic error of conical surface setting

The systematic alignment error for an ideal conical surface can be expressed mathematically as the difference between the longer and shorter positions of the ellipse, as seen in Fig. 5:

$$\xi = a - b \tag{5}$$

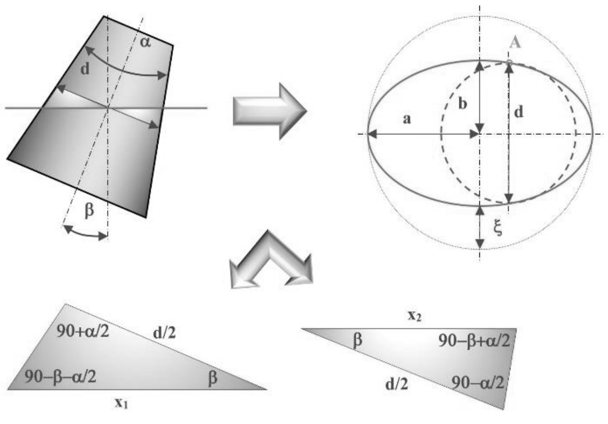

Fig. 5 *Scheme to determine the systematic error of conical surface alignment*

Where:

d…Diameter of the measured conical surface,

α…Angle of apex of the conical surface,

β…Angle of inclination of the conical surface,

a…Longer half of the ellipse (roundness profile),

b…Shorter half of the ellipse (roundness profile),

ξ…Systematic positioning error.

When calculating the longer half of the ellipse, we start from two triangles that arise when tilted (Fig. 5). We will use the sine theorem:

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma} \tag{6}$$

From the first triangle, we get:

$$x_1 = \frac{d}{2}\frac{\sin\left(90+\frac{\alpha}{2}\right)}{\sin\left(90-\beta-\frac{\alpha}{2}\right)} \tag{7}$$

From the second triangle, we get:

$$x_2 = \frac{d}{2}\frac{\sin\left(90-\frac{\alpha}{2}\right)}{\sin\left(90-\beta+\frac{\alpha}{2}\right)} \tag{8}$$

The value of the longer half is:

$$a = \frac{x_1 + x_2}{2} \tag{9}$$

$$a = \frac{d}{4}\left(\frac{\sin\left(90+\frac{\alpha}{2}\right)}{\sin\left(90-\beta-\frac{\alpha}{2}\right)} + \frac{\sin\left(90-\frac{\alpha}{2}\right)}{\sin\left(90-\beta+\frac{\alpha}{2}\right)}\right) \tag{10}$$

When calculating the shorter half-position of an ellipse, we start from a point that lies on the ellipse $A[x_A; y_A]$ (Fig. 5). Its coordinates are as follows:

$$x_A = \frac{x_1 - x_2}{2} \tag{11}$$

$$x_A = \frac{d}{4}\left(\frac{\sin\left(90+\frac{\alpha}{2}\right)}{\sin\left(90-\beta-\frac{\alpha}{2}\right)} - \frac{\sin\left(90-\frac{\alpha}{2}\right)}{\sin\left(90-\beta+\frac{\alpha}{2}\right)}\right) \tag{12}$$

$$y_A = \frac{d}{2} \tag{13}$$

Adjust the center equation of the ellipse:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \tag{14}$$

We get the relation for calculating the shorter half of the ellipse:

$$b = \frac{ya}{\sqrt{(a+x)(a-x)}} \tag{15}$$

Substituting relation (10) and coordinates (12) and (13) into relation (15) and gradual adjustment, we get:

$$b = \frac{d}{4}\frac{\frac{\sin\left(90+\frac{\alpha}{2}\right)}{\sin\left(90-\beta-\frac{\alpha}{2}\right)} + \frac{\sin\left(90-\frac{\alpha}{2}\right)}{\sin\left(90-\beta+\frac{\alpha}{2}\right)}}{\sqrt{\frac{\sin\left(90+\frac{\alpha}{2}\right)\,\sin\left(90-\frac{\alpha}{2}\right)}{\sin\left(90-\beta-\frac{\alpha}{2}\right)\,\sin\left(90-\beta+\frac{\alpha}{2}\right)}}} \tag{16}$$

The relation for the systematic setting error for an ideal conical surface is obtained by substituting equations (10) and (16) into equation (5):

$$\xi = \frac{d}{4}\left(\frac{\sin\left(90+\frac{\alpha}{2}\right)}{\sin\left(90-\beta-\frac{\alpha}{2}\right)} + \frac{\sin\left(90-\frac{\alpha}{2}\right)}{\sin\left(90-\beta+\frac{\alpha}{2}\right)} - \frac{\frac{\sin\left(90+\frac{\alpha}{2}\right)}{\sin\left(90-\beta-\frac{\alpha}{2}\right)} + \frac{\sin\left(90-\frac{\alpha}{2}\right)}{\sin\left(90-\beta+\frac{\alpha}{2}\right)}}{\sqrt{\frac{\sin\left(90+\frac{\alpha}{2}\right)\,\sin\left(90-\frac{\alpha}{2}\right)}{\sin\left(90-\beta-\frac{\alpha}{2}\right)\,\sin\left(90-\beta+\frac{\alpha}{2}\right)}}}\right) \tag{17}$$

The relationship can be simplified using the following:

$$\sin(90-\alpha) = \cos\alpha \tag{18}$$

$$\cos(-\alpha) = \cos\alpha \tag{19}$$

We get the resulting relationship:

$$\xi = \frac{d}{4}\left(\cos\left(\frac{\alpha}{2}\right) - \sqrt{\cos\left(\beta+\frac{\alpha}{2}\right)\,\cos\left(\beta-\frac{\alpha}{2}\right)}\right)\left(\frac{1}{\cos\left(\beta+\frac{\alpha}{2}\right)} + \frac{1}{\cos\left(\beta-\frac{\alpha}{2}\right)}\right) \tag{20}$$

Equation (20) is used to calculate the systematic error in setting the outer and inner conical surfaces. This error depends on the diameter d (at the measuring point of the conical surface), on the apex angle α and the angle of inclination β of the measured conical surface, as shown in Fig. 6. In this case, the apex angle $\alpha = 50°$ was taken into account.

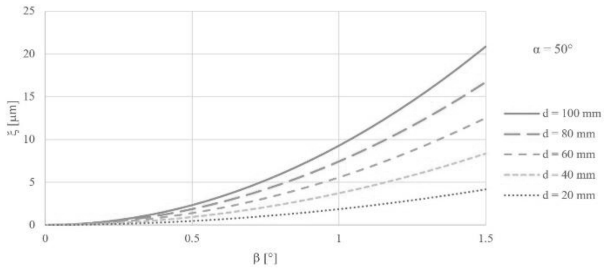

Fig. 6 *Error size ξ for different inclinations β and diameters d of the measured conical surfaces*

Fig. 7 presents a graphical dependence for calculating the systematic error of conical surface alignment on the apex angle of the cone. The diameter of the conical surface (at the measuring point) d = 60 mm was calculated in the graph. As the apex angle of the conical surface increases, the value of the error caused increases.

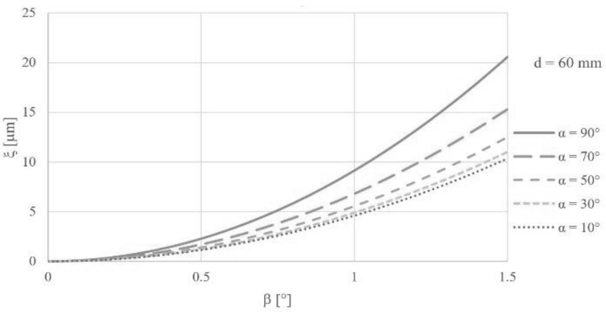

Fig. 7 *Error size ξ for different inclinations β and angles α of the measured conical surfaces*

5 Comparison of calculated and measured values

Roundness was measured on real surfaces to verify the derived mathematical relationships. Measurements were made on a Rondcom 60A (Accretech, Japan, Fig. 8). The measured part was placed on the measuring table of the device. It was centered and aligned so that the measurement plane was perpendicular to the part axis, in automatic mode. Then the roundness was measured. The parameters for roundness measurement and evaluation were as follows:

- Rotation precision: (0.02+6H/10000) μm
- Number of points measured by roundness measurement: 3600
- Method of roundness evaluation: MZC (The Minimum Zone Circle) [14, 15]
- The filter used to evaluate the roundness: Gauss low 150 UPR
- Measurement speed: 4.0 min^{-1}

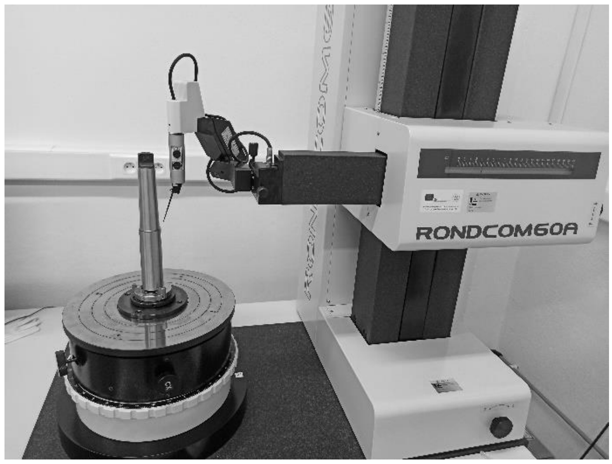

Fig. 8 *Rondcom 60A*

The round gauge measuring table allows for tilting of the part. Subsequently, the component was tilted, the measuring plane not perpendicular to the axis of it. The magnitude of the tilt angle was measured by touching a feeler gauge. The part was centered and measured the roundness of the inclined surface. The change in roundness was observed:

- Measured error = roundness $_{\text{of the inclined surface}}$ - roundness $_{\text{of the leveled surface}}$

The measurement was performed on two surfaces (Tab. 1):

- Cylindrical surface: diameter d = 36.05 mm, measured profiles are presented in Fig. 9 - a, b, c
- Conical surface: diameter at the measuring point d = 30.15 mm, apex angle of the cone α = 2.992°, the measured profiles are presented in Fig. 9 - d, e, f

Tab. 1 *Measured and calculated values*

Surface	*Tilt β [°]*	*Roundness [μm]*	*Measured error[μm]*	*Calculated error [μm]*
cylinder	*0*	*3.116*	*0*	*0*
cylinder	*0.527*	*3.316*	*0.200*	*0.762*
cylinder	*1.031*	*5.378*	*2.262*	*2.919*
cone	*0*	*4.399*	*0*	*0*
cone	*0.715*	*5.562*	*1.163*	*1.175*
cone	*1.039*	*6.764*	*2.365*	*2.481*

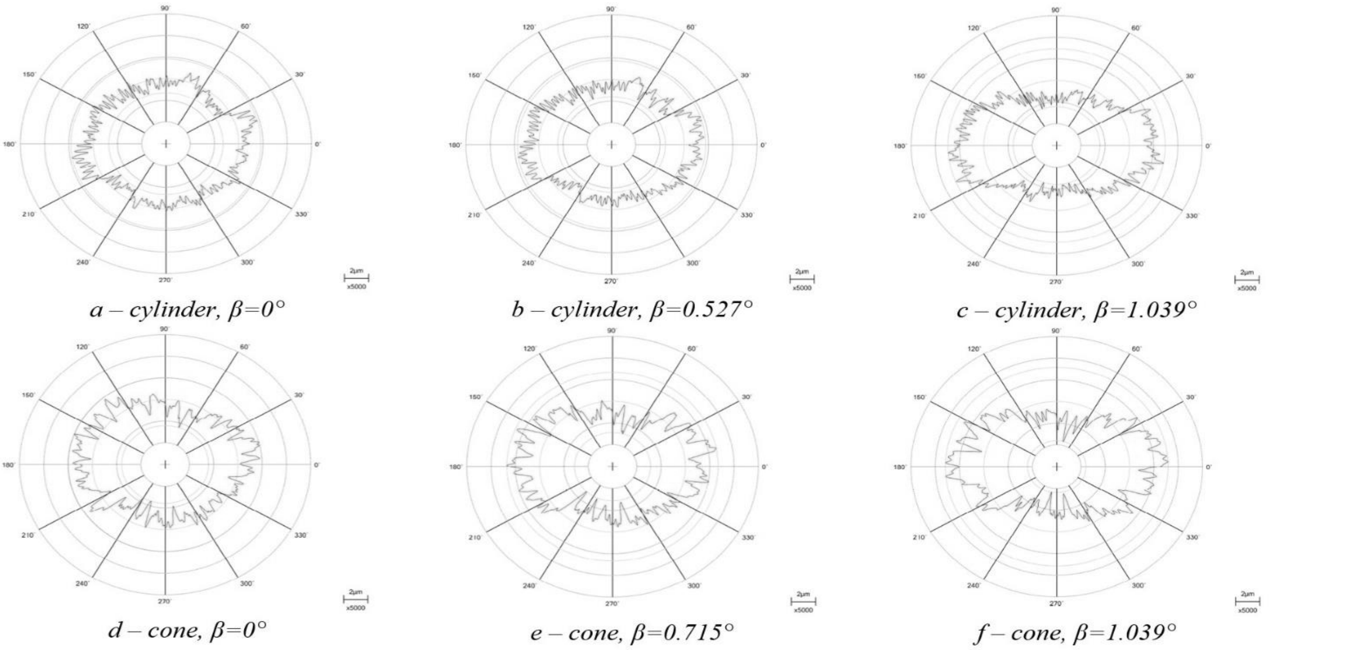

Fig. 9 *Roundness profiles measured on a cylindrical and conical surface (β = angle of inclination)*

Fig. 10 shows a graphical comparison of the calculated error and the measured roundness difference. The cylindrical surface (d = 36.05 mm) there shows the calculated error in blue. The blue circles show the differences in roundness between the inclined and level surfaces (measured error). This error in tilting the measured area was slightly smaller than the theoretically calculated error. But it has the same trend. The increasing angle of inclination increases the calculated error, and the effective measured increase in roundness increases. The calculated error for the conical surface (d = 30.15 mm, a = 2.992°) is shown in red. The red squares show the differences in roundness between the inclined and level surfaces (measured error). With this conical surface, the measured errors are very close to the theoretically calculated values.

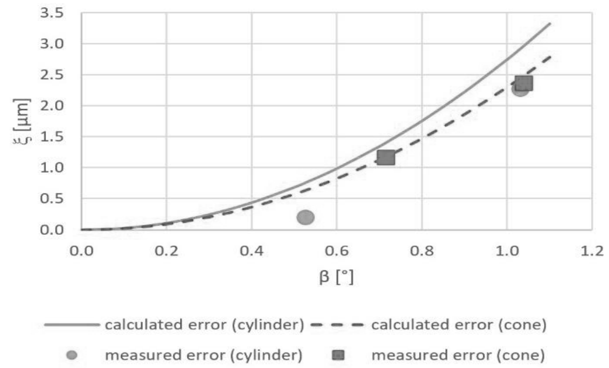

Fig. 10 *Comparison of calculated and measured errors*

6 Conclusion

We can state that the theoretically derived mathematical relations for the cylindrical and conical surfaces correspond to the real measured values.

Cylindrical surface:

- The value of the positioning error ξ depends on the inclination of the measured area β and its diameter d
- As the angle of inclination of the measured area β increases, the calculated error ξ also increases
- As the diameter of the measured area d increases, the calculated error ξ also increases

Conical surface:

- The value of the positioning error ξ depends on the inclination of the measured surface β, on its diameter d, and the apex angle of the cone α
- As the angle of inclination of the measured area β increases, the calculated error ξ also increases
- As the diameter of the measured area d increases, the calculated error ξ also increases
- As the angle of the apex measured area α increases, the calculated error ξ also increases

The inclination of the measured area β negatively affects the precision of the roundness measurement. It also affects the measured circular profile: after tilting, the roundness profile is elliptical. The measured results confirmed this statement: the value of roundness increased during tilting. However, when measuring, the inclination of the measured area can affect the accuracy of the measurement and vice versa - tilting can reduce the measured roundness. It is forming in the surface where the roundness profile is elliptical, the inclination is in the direction of the smaller half of the ellipse. Such an example is shown schematically in Fig. 11.

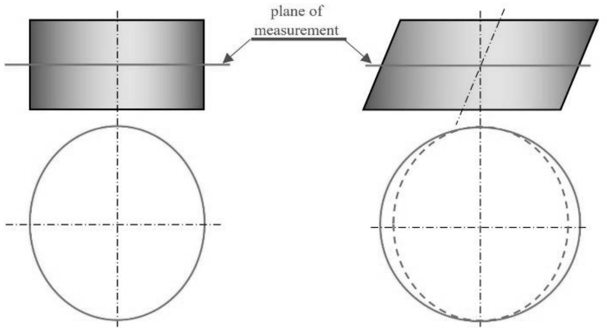

Fig. 11 *Reduction of roundness by tilting the measured area*

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