Model of geometrical deviations in milling with three error sources

Andrea Corrado (0000-0002-5418-5018), Wilma Polini (0000-0002-6839-3889)
Dipartimento di Ingegneria Civile e Meccanica, Università degli Studi di Cassino e del Lazio Meridionale, 03043 Cassino, Italy. E-mail: andreacorrado13@gmail.com, polini@unicas.it.

Article abstract
The milling process is widely used industrially because it allows manufacturing surfaces with many different and complex shapes [1]. High-value applications require control of the quality and the manufacturing signature of the products because they affect their performances in exercise [2-3]. Achieving this aim is not so easy. Roughness is the most industrially used indicator to describe surface quality; however, it is not enough to perform evaluations on workpiece performances [4]. Therefore, it is needed to add an indicator of surface geometrical deviation, such as flatness and orientation.

At the same time having a model to foresee the geometrical deviations, the surface roughness and the surface morphology of a manufactured part may help production engineers [5]. Their experience guides their choices of the machine, the tools and the process parameters.

The most studied milling process was the peripheral one because it involves a 2D study. The effects of the cutting speed, the tool wear and vibration on the machined surface topography were studied in [6]. The kinematics of the machine tool was put together the geometry of the tool to foresee the wall surface [7]. A methodology to correct the relative error motion between the tool and the workpiece was presented by [8]; it uses reference parts as methodological standards. A system able to predict surface texture from highly sparse learning data was developed by [9].

The effect of cutter runouts on roughness was investigated by [10]. This model was extended by [11] to consider flank wear. Some researchers studied the influence of tool deflection on the machined surface profile [12-13]. Other researchers correlated cutting vibrations and the quality of machined surface [14-16]. Lartigue et al. investigated the influence of CAM parameters on form deviation and roughness of the obtained surface [17]. Zhang and Guo proposed an error compensation method of the tool path [18]. A model to simulate the geometrical deviations in the midplane position of cavities machined by milling was studied by [19].

A force model combined with a kinematic model was used to evaluate the topography in face milling operations [20]. Altintas and Engin developed a parametrical model of each cutting edge profile to move during milling that allows estimating the cutting forces and vibrations [21]. The kinematic interference between inserts and material was translated in a model by [22]. The surface roughness was estimated through a geometrical model of face milling by squared inserts [23]. Torta et al. proposed a model to foresee the roughness in face milling that takes into account different tool geometries and the relative alignment/mounting errors [24]. This model was validated on a high-feed milling operation. Cutting force and temperature were studied as a function of the milling insert type in
[25] and of the milling parameters in [26-27] in face milling of steel workpieces. The best machining strategy to improve the accuracy of the 3D shapes is studied in [28].

All the previously described approaches model only one source of machining error, that due to machine tool. However, in addition to the machine tool error, there may be a misalignment between the workpiece and the machine reference systems. This misalignment affects the location and the orientation of the surface to the machine. It depends on the deviations from nominal of the contact points among the used locators and the part surfaces, that are connected with the deviation from nominal of locators’ positions and the shape deviations of the workpiece surfaces into contact with those locators.

In the literature, many are the papers that evaluate how fixture affects the part deviation using screw theory [29-30], geometric perturbation techniques [31], stability index [32], small-displacement torsor concept [33]. Optimization techniques were used for the minimization of localization error or feature variation [34-35]. Genetic Algorithms were used to define the layout of the fixture in [36]. Li et al. considered the influence of the deviations of the locating system, of the machine tool together and the cutting parameters on the quality of a milled surface [37]. However, even in these cases, the approaches proposed by the literature involve only one or at most two sources of error in machining operations.

This paper increases knowledge on machining by considering the contributions of three sources of errors on the quality of a milled surface. The errors connected with the used locator system, to the part surfaces and machine tool, were modelled. It presents a framework inside which it is possible to model differently the contribution to milled part deviation due to each error sources.

The scope of this work is to kinematically correlate the geometrical error (or manufacturing error) of a surface obtained through a face milling with the deviations in locators’ positions, the part shape deviations surfaces that are in contact with the locators and the volumetric error of the machine tool.

In previous works, the authors statistically predicted the deviation from nominal of a hole or a hole pattern position connected with the deviations from nominal of the positions of six locators in the 3-2-1 locating scheme [38-39]. Moreover, the deviations in the fixture position were combined with the machine tool error [40], with the form deviation of the workpiece datum surface [41] or with both the machine error and the datum form deviation [42].

In this work, a model was developed and implemented by Matlab® software that considers the deviations from the machine reference frame of the part reference frame due to the locators, datum surfaces and machine tool. In this way, it was possible to avoid a lot of experimental tests and, therefore, time, energy and materials involved by them. The proposed approach was applied to two milling processes.

In §2, the model is described, while in §3, it is applied to two milling processes characterizing two different configurations of locators.

2 Materials and methods

The nominal surface obtained through a face milling process is represented by a skin model that means through point clouds. The deviations in the location of the points on the milled surface and that on the nominal surface represent the geometric errors. They are modelled as a function of locators, workpiece and machine tool.

The deviations from the nominal of the workpiece reference frame are connected with the machining fixture that is made by reference elements. In this study, the 3-2-1 locator scheme was considered, whereas six locators influence the location of the three mutually orthogonal planes of the reference frame. Locators’ misalignment involves a deviation from the nominal of the workpiece reference frame and, therefore, of the milled surfaces, such as flatness or an orientation error on face milled surfaces. The six locators have eighteen coordinates that define their position of which only six ones influence significantly the machining error [38]. Each of the six coordinates followed a probability density function.

Every manufacturing process leaves on the machines surfaces a signature that typically involves peculiar deviation from nominal design [43-45]. A Simultaneous Autoregressive Model of first-order SAR(1) was used to consider the signature on the datum planes contacting the six locators because it can take into account correlated phenomena in easy ways. A set of evenly distributed points was used to simulate the three planes into contact with the six locators, while the workpiece reference frame was simulated through the six points on the three datum planes nearest to the locators.

A machine tool is a kinematic chain of components, whose geometric deviations may be modelled using a transformation matrix and the machine tool resulting error is given by multiplying the transformation matrices according to the order of the kinematic chain [43]. This paper considers the static volumetric of the machine tool.

The plane to be milled was represented by 3D points, that are arranged in a grid along with the two x-y directions and the distance between two consecutive points is equal to the nominal radius of the cutter along each of the two x-y directions (see Fig. 1a); thus, a matrix of m x n 3D points represents the skin model of the nominal plane
(see Fig. 1b). The coordinates of each point belonging to the milled surface in the workpiece reference frame (WRF) are estimated starting from the coordinates in the machine reference frame (MRF) and switching to the locator reference frame (LRF) and then to the workpiece reference frame through the laws of robot kinematic, as shown in Fig. 2. Fig. 2 shows the geometrical deviation of a milled surface due to the synergistic effect of locators (first arrow), the flatness of datum surface (second arrow) and volumetric error of the machine tool (third arrow). Finally, the coordinates of each point in the WRF are corrected with the error of the machine tool to have the points on the milled surface.

Fig. 1 a) Set of diameters of the cutter during the milling, b) Skin model shape of a milled plane

Fig. 2 Scheme of the proposed model
The method of least squares was used to estimate an approximate plane from the coordinates of the points on the milled surface. The flatness was evaluated as the sum of the distances from the least-squares plane of the two points that are the most distant. To evaluate the deviation of the milled surface orientation, the angle between the normals of the top and bottom planes was calculated.

The angle between the normals of top and bottom planes and the flatness of the actual milled surface represent the functional requirements, as shown in Fig. 3 on the right. Fig. 3 on the left shows the part deviation connected with both locators and datum surfaces.

The workpiece position was built through the six locators shown in Fig. 4. The coordinates of the i-th locator are represented by \( p_i(x_i, y_i, z_i) \). The proposed model express the coordinates of the points belonging to the milled surface in the workpiece reference frame. It takes as input the dimension of the workpiece, the nominal locator configuration, the diameter of the cutter, the number and nominal coordinates of the points on the top surface and the manufacturing signature of the milling process.

\[ \begin{align*}
\mathbf{P}_i' &= 0R_p^{-1} \ast \mathbf{P}_i = \begin{bmatrix} \mathbf{R} & \mathbf{t}_i \end{bmatrix} \begin{bmatrix} 0 \mathbf{r} \end{bmatrix} \ast \mathbf{P}_i \\
\end{align*} \]

with the rotational matrix \( \mathbf{R} \), the translation vector \( \mathbf{t}_i \), and \( \mathbf{0r} \) is a zeros vector \([3x1]\). In detail

\[ R = \begin{bmatrix} n_x^T \\ n_y^T \\ n_z^T \end{bmatrix} \]  \( n_{x} = \frac{p_{45} \ast v_{1}}{||p_{45} \ast v_{1}||} \)  \( p_{45} = p_{5} - \)
2.2 Contribution of form deviation on workpiece datum surfaces

The form deviation or the manufacturing signature (f) of each plate surface contacting the locators was simulated through a spatial autoregressive (SAR) model [47]:

\[
f = (I - \rho V)^{-1} \epsilon
\]

with the identity matrix \(I\), the spatial autoregressive parameter \(\rho\) (whose value is 0.9), the spatial weighting matrix \(V\) and the white noise \(\epsilon \sim (0, \sigma^2 I)\). The matrix \(W\) is estimated as:

\[
w_{lj} = \frac{d_{lj}}{\sum_{k \neq j} d_{kj}}
\]

where \(d_{lj}\) is the Cartesian distance calculated between the \(P_l\) and the \(P_j\) points, and \(I_{lj}\) is evaluated as:

\[
I_{lj} = \begin{cases} 
1, & \text{if point } l \text{ and } j \text{ belong to a same triangle} \\
0, & \text{otherwise}
\end{cases}
\]

Given a set of samples \(\{(x_l, y_l, z_l)\}_{l=1}^m\) from Eq. (10) to represent the three surfaces of the plate into contact with the 3-2-1 locators, Once obtained some samples \(\{(x_l, y_l, z_l)\}_{l=1}^m\) from Eq. (10) on the three plate surfaces, it is needed to determine a set of six points, \(R, S, T, U, W, Z\) so that

\[
\begin{align*}
M_i &= \min(|R - P_1|, |S - P_2|, |T - P_3|) \\
M_j &= \min(|U - P_4|, |W - P_5|) \\
M_k &= \min(|Z - P_6|)
\end{align*}
\]

Therefore, it is possible to express the coordinates of the points \(P_i'(x'_i, y'_i, z'_i)\) of the top plane of the workpiece from the LRF to the part reference frame (PRF):

\[
P_i' = R_p^{-2} \cdot P_i' = \begin{bmatrix} R' & t'_i & P_i' \end{bmatrix}
\]

with \(R'\) is the rotational matrix which allows passing by the LRF to PRF, while \(t'_i\) is the vector describing the LRF origin position referred to the PRF, and \(o^T\) is a zeros vector \([3 \times 1]\). In detail

\[
R' = \begin{bmatrix} n'_x^T & n'_y^T & n'_z^T \end{bmatrix}
\]

\[
U'W' = W' - E'
\]

\[
U' = 0R_p^{-1} \cdot U
\]

\[
W' = 0R_p^{-1} \cdot EW
\]

\[
Z' = 0R_p^{-1} \cdot Z
\]

\[
n'_x = \frac{D' E' \cdot v_{1'}}{\|D' E' \cdot v_{1'}\|}
\]

\[
U = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}
\]

\[
R' T' = T' - R'
\]

\[
R' S' = S' - R'
\]

\[
R' = 0R_p^{-1} \cdot R
\]

\[
S' = 0R_p^{-1} \cdot S
\]
\[ T' = _0^1 R_p^{-1} * T \]  
\[ t'_i = \begin{bmatrix} n_x' * W' \\ n_y' * Z' \\ n_z' * R' \end{bmatrix} \]  
(29)  
(30)

2.3 Contribution of the volumetric error in the machine tool

It is possible to estimate the position error \( \Delta p \) of the mill tip and the direction error \( \Delta d \) of the tool axis as:

\[
[\begin{array}{cccccccc}
1 & 1 & 1 & 0 & 0 & 0 & \ldots \\
0 & 0 & 0 & 0 & 0 & 0 & \ldots \\
z-l & z-l & -l & -y & 0 & 0 & \ldots \\
0 & 0 & 0 & 0 & 0 & 0 & \ldots \\
0 & 0 & 0 & 0 & 0 & 0 & \ldots \\
0 & 0 & 0 & 0 & 0 & 0 & \ldots \\
1 & 1 & 1 & 0 & 0 & 0 & \ldots \\
0 & 0 & 0 & 0 & 0 & 0 & \ldots \\
0 & 0 & 0 & 0 & 0 & 0 & \ldots \\
-1 & -1 & -1 & 0 & 0 & 0 & \ldots \\
0 & 0 & 0 & 0 & 0 & 0 & \ldots \\
0 & 0 & 0 & 0 & 0 & 0 & \ldots \\
0 & 0 & 0 & 0 & 0 & 0 & \ldots \\
0 & 0 & 0 & 0 & 0 & 0 & 1 \\
\end{array}]
\begin{bmatrix}
\delta_x(x) \\
\delta_y(x) \\
\delta_z(x) \\
\delta_x(y) \\
\delta_y(y) \\
\delta_z(y) \\
\delta_x(z) \\
\delta_y(z) \\
\delta_z(z) \\
\epsilon_x(x) \\
\epsilon_x(y) \\
\epsilon_x(z) \\
\epsilon_y(x) \\
\epsilon_y(y) \\
\epsilon_y(z) \\
\epsilon_z(x) \\
\epsilon_z(y) \\
\epsilon_z(z) \\
\end{bmatrix}
= C d
\]

with the translation \( \delta \) and rotation \( \epsilon \) errors (e.g. \( \epsilon_z(x) \) is the rotational error around the \( z \)-axis during the translation along the \( x \)-axis), that follow Gaussian \( N(0, \sigma^2_p) \) or \( N(0, \sigma^2_d) \) distributions respectively. Therefore, the \( [\Delta_p \Delta_d] \) vector is distributed according to with a multivariate Gaussian distribution, with null means and covariance matrix \( \Sigma \). The covariance 18x18 matrix \( \Sigma \) is diagonal with the first nine diagonal elements equal to \( \sigma^2_p \) and the remaining diagonal elements equal to \( \sigma^2_d \).

2.4 Coordinates of points on the milled surface

To the coordinates in the PRF was added the machine tool volumetric error to obtain the coordinates of the i-th point \( P_i'' \) of the milled top surface in the PRF:

\[ P_i'' = P_i' + l k'' \]  
(35)

where \( l \) is the depth of cut.

From equation (16):

\[ P_i'' = _1^2 R_p^{-2} \left[ _0^1 R_p^{-1} (P_i + \Delta_p) \right] \]  
(33)

The mill tool direction in the PRF is given by

\[ k'' = _1^2 R_p^{-2} \left[ _0^1 R_p^{-1} (k + \Delta_d) \right] \]  
(34)

where \( k \) is the mill tool direction in the machine tool reference frame.

The distances of the two points more distant from the plane connected with the least-squares method were used to estimate the flatness.

3 Discussion of the results

A plate of 100 mm x 120 mm with a thickness of 60 mm was considered. The locators are placed according to a Gaussian probability density function, centred on the nominal position and characterized by a standard deviation...
Their nominal positions were \( p_1(95, 70, 0) \), \( p_2(12.5, 117.63, 0) \), \( p_3(12.5, 22.37, 0) \), \( p_4(0, 25, 5) \), \( p_5(0, 115, 5) \), \( p_6(40, 0, 5) \), as shown in Fig. 4a. The machine tool volumetric error was characterized by \( \sigma_p = 0.01 \text{ mm} \) and \( \sigma_d = 0.01 \times \frac{\pi}{180} \text{ mm} \). The SAR model had a white noise \( \epsilon \sim (0, \sigma^2 I) \) with \( \sigma=0.01 \text{ mm} \) and a spatial autoregressive parameter \( \rho=0.9 \). The deviation of the points on the first datum along a direction parallel to the X-axis, that of the points on the second datum along a direction parallel to the Y-axis and that of the points on the third datum along a direction parallel to the Z-axis followed a uniform probability density function with \( \sigma=0.001 \text{ mm} \). These values were taken from the literature on the locating equipment, the machine tools and the Geometric Dimensioning and Tolerancing (GD&T). Fig. 5 shows one example of simulated milled surface.

The considered performance indicators are the flatness of the milled plane and the angle between the milled surface and the bottom surface. The obtained numerical results are shown in Figure 6 \( (L = \text{locator error}, M = \text{machine tool volumetric error}, F = \text{datum form deviations}, L+M = \text{locator error+volumetric error}, L+F = \text{locator+datum flatness}, F+M = \text{datum flatness+volumetric error}, L+F+M = \text{locator error+volumetric error+datum flatness}) \). The considered number of Monte Carlo simulation runs was 100,000. As it can be seen from the data, the volumetric error of the machine tool volumetric error is the only factor that influences the flatness, while the angle is affected by the locators’ deviations and the datum form deviations. The obtained results are aligned with what happens. The flatness indicates the dispersion of the surface points around a least-squares plane that has an orientation different from the nominal, the flatness is due to the volumetric error of the machine tool, while the orientation of the least-squares plane depends on the locator positions and datum surface deviations.

A further positioning of the locators was taken into account: \( p_1(95, 60, 0) \), \( p_2(5, 111.96, 0) \), \( p_3(5, 8.04, 0) \), \( p_4(0, 5, 5) \), \( p_5(0, 115, 5) \), \( p_6(95, 0, 5) \), see Fig. 4b. The other parameters are kept unchanged. The results are shown in Fig. 6; they are similar to those previously described.

An analysis of variance (ANOVA) was performed to evaluate the influence of locators’ deviations, datum form deviations and volumetric error of machine tool on angle values; the results are shown in Table 1. The datum form and locators’ deviations significantly affect the angle results; their contributions are 91% and 9% respectively. The same analysis was performed on flatness too (see Table 2) and the unique significant contribution is that of machine tool volumetric error equal to 93%, the locator configuration has an influence of 3% and the datum flatness of 4%.

**Fig. 5 Simulated milled surface (errors amplified 30 times)**

4 Conclusions

This work presents kinematic modelling to simulate the effect of three sources of deviations, i.e. locators, datum surfaces contacting locators and machine tool, on the geometrical error of a surface manufactured by face milling. Four steps were implemented: (1) evaluate how the deviations from nominal of locators’ positions affect the position and orientation of workpiece reference frame; (2) estimate how the form deviations of datum surface contacting the locators affect the position and orientation of workpiece reference frame; (3) define the machine tool volumetric error; (4) combine all the contributions.

The proposed model is based on robotic concepts of kinematic chains whose correctness was demonstrated by the literature. The proposed model considers in a simple and effective model three contributions to geometric deviations of a milled surface, thus overcoming the state of the art.

It was used on two parts that are characterized by two different configurations of the locators. From the results, it is clear how the proposed model easily allows estimating the deviation of the milled surface due to the combined effect of three error sources. The use of the proposed model allowed to save time, energy and material involved...
by an experimental approach.
Future works aim to define the values of the parameters connected with the three sources of deviations affecting a milled surface through experimental tests.

Acknowledgement
*This research did not receive any specific grant from funding agencies in the public, commercial or not-for-profit sectors.*

![Fig. 6 Results for the first (Case 1) and second (Case 2) configuration of the locators](image)

<table>
<thead>
<tr>
<th>Possible deviations</th>
<th>DF</th>
<th>SS</th>
<th>MS</th>
<th>Contribution</th>
<th>F</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Locators</td>
<td>1</td>
<td>0.0002897</td>
<td>0.0006048</td>
<td>9%</td>
<td>1167.30</td>
<td>0.000</td>
</tr>
<tr>
<td>Machine tool</td>
<td>1</td>
<td>0.0000012</td>
<td>0.0000012</td>
<td>0%</td>
<td>2.41</td>
<td>0.129</td>
</tr>
<tr>
<td>Datum</td>
<td>1</td>
<td>0.0030479</td>
<td>0.0027370</td>
<td>91%</td>
<td>5283.05</td>
<td>0.000</td>
</tr>
<tr>
<td>Locators + Datum</td>
<td>1</td>
<td>0.0000030</td>
<td>0.0000020</td>
<td>0%</td>
<td>3.83</td>
<td>0.058</td>
</tr>
<tr>
<td>Error</td>
<td>37</td>
<td>0.0000192</td>
<td>0.0000005</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>41</td>
<td>0.0033611</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Tab. 2 ANOVA of flatness results

<table>
<thead>
<tr>
<th>Possible deviations</th>
<th>DF</th>
<th>SS</th>
<th>MS</th>
<th>Contribution</th>
<th>F</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Locators</td>
<td>1</td>
<td>0.002336</td>
<td>0.0000000</td>
<td>3%</td>
<td>3.53</td>
<td>0.068</td>
</tr>
<tr>
<td>Machine tool</td>
<td>1</td>
<td>0.078735</td>
<td>0.065613</td>
<td>93%</td>
<td>66693208.24</td>
<td>0.000</td>
</tr>
<tr>
<td>Datum</td>
<td>1</td>
<td>0.003288</td>
<td>0.0000000</td>
<td>4%</td>
<td>3.53</td>
<td>0.068</td>
</tr>
<tr>
<td>Datum + machine tool</td>
<td>1</td>
<td>0.0000000</td>
<td>0.0000000</td>
<td>0%</td>
<td>3.53</td>
<td>0.068</td>
</tr>
<tr>
<td>Locators + machine tool</td>
<td>1</td>
<td>0.0000000</td>
<td>0.0000000</td>
<td>0%</td>
<td>3.53</td>
<td>0.068</td>
</tr>
<tr>
<td>Error</td>
<td>36</td>
<td>0.0000000</td>
<td>0.0000000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>41</td>
<td>0.084359</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### References


