

The Role of Modelling of Road Unevennesses in Vehicle Dynamics

Stefan Segla¹, Jan Kampo²

¹Faculty of Mechanical Engineering, TU in Košice, Letná 9, 042 00 Košice, e-mail: stefan.segla@tuke.sk

²Faculty of Mechanical Engineering, J. E. Purkyne University in Usti nad Labem, Pasteurova 3334/7, 400 96 Ústí nad Labem, jan.kampo@ujep.cz

It is well known that the dynamic behaviour of a vehicle is affected by the design parameters of its suspension system, especially by the stiffnesses of the suspension springs, damping coefficients and tire stiffnesses. Another important factor influencing vehicle vibration is kinematic excitation caused by uneven roads. It may significantly affect the comfort of the driver and passengers, safety of the ride and relative displacements between the sprung and unsprung masses. The paper presents mathematical models of both deterministic and random road unevennesses and numerical simulation of vertical dynamics of planar vehicle models with kinematic excitation caused by these road unevennesses. When examining transient phenomena, standardized obstacles according to STN 30 0560, resp. EU Directive 85/3/EW6-III are applied. Random unevennesses can be obtained experimentally, or generated by the Shinozuka method to create the mathematical model of an uneven road with a specified power spectral density. Actual vehicle prototypes need to be tested on test circuits with different surfaces.

Keywords: road, modelling, vehicle dynamics, optimization, stochastic excitation

1 Introduction

The vehicle is exposed to different types of excitation while driving. In particular, it is a kinematic excitation caused by road unevenness, but also by excitation from unbalanced engine masses and other moving parts of the vehicle. Furthermore, we will only consider kinematic excitation caused by road unevennesses. These can be divided into two basic types: deterministic and stochastic unevennesses, as described, for example, by Nigam (1994) [3].

Poor roads are among the most important sources of vehicle vibration. It significantly influences crew comfort, cargo safety and vehicle durability. They also affect the safety of the ride and the amount of space required between the sprung and unsprung mass. That is why they need to pay attention to their modelling [11].

When designing vehicles it is important for the vehicle to be tested for the various types of roadway where its functionality is expected. This testing is needed both by computer simulation and by testing prototype vehicles on the test circuit [8].

2 Deterministic obstacles

Deterministic obstacles are used to assess the transient processes of real vehicles, see, for example, Můčka [2], according to STN 30 0560 [6]. This standard defines three types of deterministic obstacles, Fig. 1:

- a cylindrical unevenness with a length of 0.5 m and a height of 3 cm, curve 1 in Fig. 1a
- a cylindrical unevenness with a length of 0.5 m and a height of 6 cm, a curve 2 in Fig. 1a,
- a trapezoidal pit with a depth of 6 cm and a length of horizontal trapezoidal sides of 0.415 m (bottom) and 0.6 m (top), curve 3 in Fig. 1a.

One of the often used retarders is given by a 0.44 m cylindrical unevenness with a height of 5 cm (curve 4 in Figure 1a). The EU standard 85/3 / EW6-III [1] defines a trapezoid with a base length of 5.8 m and a height of 8 cm, while the length of the shorter horizontal side is 0.8 m and the vehicle suspension properties are analysed at a speed of 5 km / h (curve 6 in Figure 1b). The so-called roller blind is approximated by a 3 cm amplitude sinusoidal signal with a wavelength of 0.5 m (curve 5 in Figure 1b).

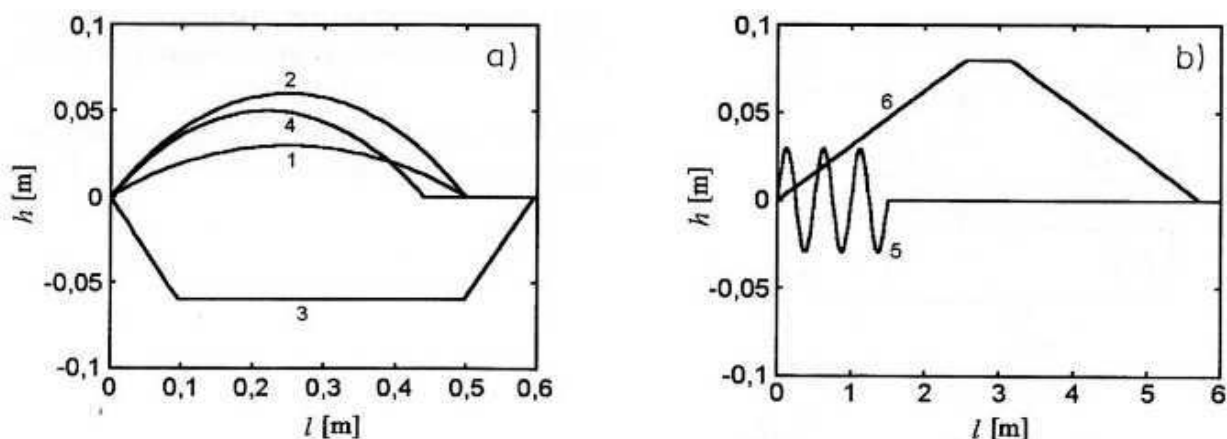


Fig. 1 Obstacles used to investigate transient events

Normalized obstacles, however, are suitable for detecting the properties of suspensions of real vehicles, where the tire and its compliance plays a significant role. For idealized tire contact with the road, these standard obstacles for numerical simulations are not suitable [10].

A suitable non-normalized obstacle for numerical simulation of vehicles with idealized tire-to-road contacts is the deterministic hurdle of the so-called „hat“ shape, which to some extent respects the actual shape of the tire, Fig. 2. It is used, for example, in Volek et al. [7].

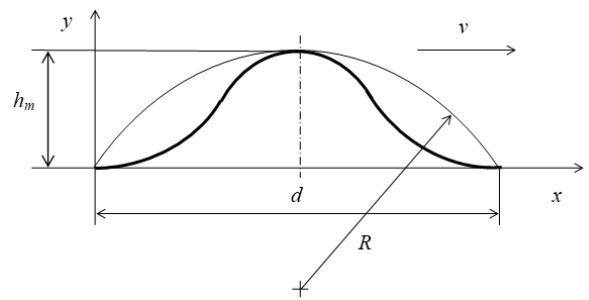


Fig. 2 Deterministic obstacle of the hat shape

Mathematical model of the obstacle, depending on the track x is described by equation

$$u(x) = \frac{h_m}{2} \left(1 - \cos \frac{2\pi}{d} x \right), \quad \text{for } 0 \leq x \leq d, \quad \text{otherwise } u(x) = 0, \quad (1)$$

where

R is the radius of the circumscribed circle (Figure 2),
 d is the length of the obstacle and h_m its height.

In numerical simulation, the time dependence of the instantaneous obstacle height will be important

$$u(t) = \frac{h_m}{2} \left(1 - \cos \frac{2\pi}{T} t \right), \quad \text{for } 0 \leq t \leq T, \quad \text{otherwise } u(t) = 0, \quad (2)$$

where

$$T = \frac{d}{v}, \quad t = \frac{x}{v}, \quad (3)$$

while v is considered vehicle speed and T is the obstacle transit time [8].

Actual vehicle prototypes need to be tested on test circuits with different surfaces, as shown in Fig. 3.



Fig. 3 Surfaces of different test circuits (source: internet)

3 Random road inequality

Very important is computer modelling of vehicles

while driving on a stochastically uneven path. Generating such paths with prescribed power spectral density (which corresponds to paths of various kinds - from very uneven to very good quality routes) allows Shinozuka method.

Road surfaces have a random (stochastic) profile of unevenness. The height of unevenness u is generally a two-dimensional random function of position in the plane of the path, which is given by the coordinates x in the longitudinal direction and z in the transverse direction. Unevenness can be described by function

$$u = u(x, z). \quad (4)$$

One function record (4) is called realization. More records of this feature are a set of implementations. This file describes a random run, in this case the course of road unevenness. It is assumed that the random function of road inequality is a steady ergodic process, which means that the probability characteristics of this process do not depend on time and track and that only a sufficiently long realization is sufficient to determine these characteristics. The condition of steadiness and ergodicity is usually met as the observed section of the road is sufficiently long [9].

3.1 Simulation of the road unevennesses by the sum of cosine functions with prescribed power spectral density (Shinozuka method)

The basis of this method is the idea that each random stationary process can be represented by the sum of cosine functions. This method was developed in 1971 by Shinozuka. His theory can be found in Nigam [3].

Shinozuka's method can be described by the equation

$$u(t) = \sqrt{2} \sum_{k=1}^N A_k \cos(\omega'_k t + \phi_k), \quad (5)$$

where

$$A_k = \left[S_u(\omega_k) \Delta\omega \right]^{\frac{1}{2}}, \quad (6)$$

$$\omega'_k = \omega_k + \delta\omega_k, \quad (7)$$

$$\omega_k = \omega_L + \left(k - \frac{1}{2} \right) \Delta\omega, \quad (8)$$

$$\Delta\omega = \frac{\omega_U - \omega_L}{N}, \quad (9)$$

where ω_L and ω_U are the lower and upper limits for which the power spectral density $S_u(\omega)$ acquires negligible values. They are therefore given by the analysis of the particular problem solved.

Power spectral density of path unevenness $S_u(\omega_k)$ is given by equation

$$S_u(\omega_k) = \frac{v S_u(\Omega_0) \Omega_0^2}{\omega_k^2}, \quad (10)$$

where is

v - vehicle speed,

Ω_0 - reference circular frequency ($\Omega_0 = 1 \text{ m}^{-1}$),

$S_u(\Omega_0)$ - power spectral density of path unevenness

for $\Omega_0 = 1 \text{ m}^{-1}$.

In equation (5) acquires ϕ_k random values with a uniform probability distribution in the interval $[0, 2\pi]$.

In equation (7) $\delta\omega_k$ is a small random frequency that acquires values with a uniform probability distribution in the interval $\left[-\frac{\Delta\omega'}{2}, \frac{\Delta\omega'}{2} \right]$, while $\Delta\omega' \ll \Delta\omega$.

Small random frequency $\delta\omega_k$ is used to avoid the occurrence of periodicity of the generated random function.

The method makes it possible to simulate the ride along this path with the prescribed power spectral density of the road unevenness $S_u(\Omega_0)$ - at any time t is possible to calculate the height of the inequality $u(t)$ from equation (5). It is obvious that this calculation is necessary before the simulation of the vehicle dynamics itself, whereas in the simulation a displacement vector is needed $u(t_i)$ for t_i ($i=1, 2, 3, \dots$), where

$$t_i = t_{i-1} + \Delta t \quad (11)$$

and Δt depends on the integration step that is used to solve the mathematical model of vehicle dynamics.

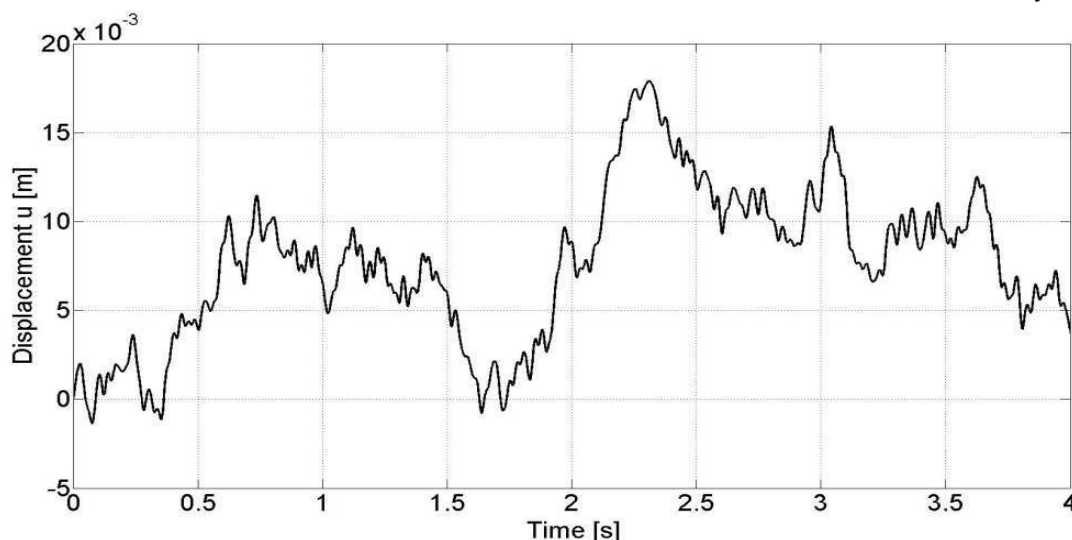


Fig. 4 Stochastic path generated by the Shinozuka method

Fig. 4 shows the stochastic path that was generated by the Shinozuka method with input parameters $S_u(\Omega_0) = 22 \cdot 10^{-6} \text{ m}^3$ (good asphalt-concrete path), $v = 20 \text{ m} \cdot \text{s}^{-1}$.

4 Planar model of vehicle

The mechanical model of a plane vehicle model is represented by Fig. 5.

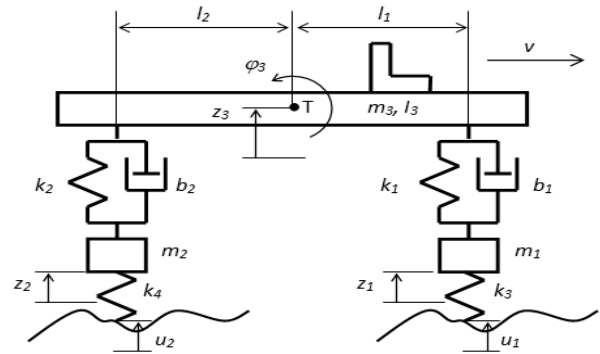


Fig. 5 Planar model of vehicle

The mathematical model derived by the d'Alembert principle is

$$m_1 \ddot{z}_1 - b_1 (\dot{z}_3 + l_1 \dot{\phi}_3 - \dot{z}_1) - k_1 (z_3 + l_1 \phi_3 - z_1) + k_3 (z_1 - u_1) = 0, \quad (12a)$$

$$m_2 \ddot{z}_2 - b_2 (\dot{z}_3 - l_2 \dot{\phi}_3 - \dot{z}_2) - k_2 (z_3 + l_2 \phi_3 - z_2) + k_4 (z_2 - u_2) = 0, \quad (12b)$$

$$m_3 \ddot{z}_3 + b_1 (\dot{z}_3 + l_1 \dot{\phi}_3 - \dot{z}_1) + b_2 (\dot{z}_3 - l_2 \dot{\phi}_3 - \dot{z}_2) + k_1 (z_3 + l_1 \phi_3 - z_1) - k_2 (z_3 - l_2 \phi_3 - z_2) = 0, \quad (12c)$$

$$I_3 \ddot{\phi}_3 + b_1 l_1 (\dot{z}_3 + l_1 \dot{\phi}_3 - \dot{z}_1) - b_2 l_2 (\dot{z}_3 - l_2 \dot{\phi}_3 - \dot{z}_2) + k_1 l_1 (z_3 + l_1 \phi_3 - z_1) - k_2 l_2 (z_3 - l_2 \phi_3 - z_2) = 0, \quad (12d)$$

A system of equations of the first order is transformed

$$\dot{z}_1 = y_1, z_1 = y_2, \dot{z}_2 = y_3, z_2 = y_4, \dot{z}_3 = y_5, z_3 = y_6, \dot{\phi}_3 = y_7, \phi_3 = y_8. \quad (13)$$

The mathematical model of the solved system as a system of ordinary differential equations of the first order, see Rahman (1991) [5]

$$\dot{y}_1 = \frac{1}{m_1} [2b_1 (y_5 + l_1 y_7 - y_1) + k_1 (y_6 + l_1 y_8 - y_2) - k_3 (y_2 - y_1)], \quad (14a)$$

$$\dot{y}_2 = y_1, \quad (14b)$$

$$\dot{y}_3 = \frac{1}{m_2} [2b_2 (y_5 - l_2 y_7 - y_3) + k_2 (y_6 + l_2 y_8 - y_4) - k_4 (y_4 - u_2)], \quad (14c)$$

$$\dot{y}_4 = y_3, \quad (14d)$$

$$\dot{y}_5 = \frac{1}{m_3} [-2b_1 (y_5 + l_1 y_7 - y_1) - 2b_2 (y_5 - l_2 y_7 - y_3) - k_1 (y_6 + l_1 y_8 - y_2) - k_2 (y_6 - l_2 y_8 - y_4)], \quad (14e)$$

$$\dot{y}_6 = y_5, \quad (14f)$$

$$\dot{y}_7 = \frac{1}{I_3} \left[-2b_1 l_1 (y_5 + l_1 y_7 - y_1) + 2b_2 l_2 (y_5 - l_2 y_7 - y_3) - k_1 l_1 (y_6 + l_1 y_8 - y_2) + k_2 l_2 (y_6 - l_2 y_8 - y_4) \right], \quad (14g)$$

$$\dot{y}_8 = y_7. \quad (14h)$$

ration when crossing the „hat“ obstacle, respectively, driving on a stochastically uneven path. The Optimization Toolbox for Use with Matlab [4] was used for numerical analysis and optimization of the vehicle mechanical model parameters.

Fig. 6 to 9 show a progressive development of the centroid T of the vehicle sprung mass Fig. 5, and its acceleration

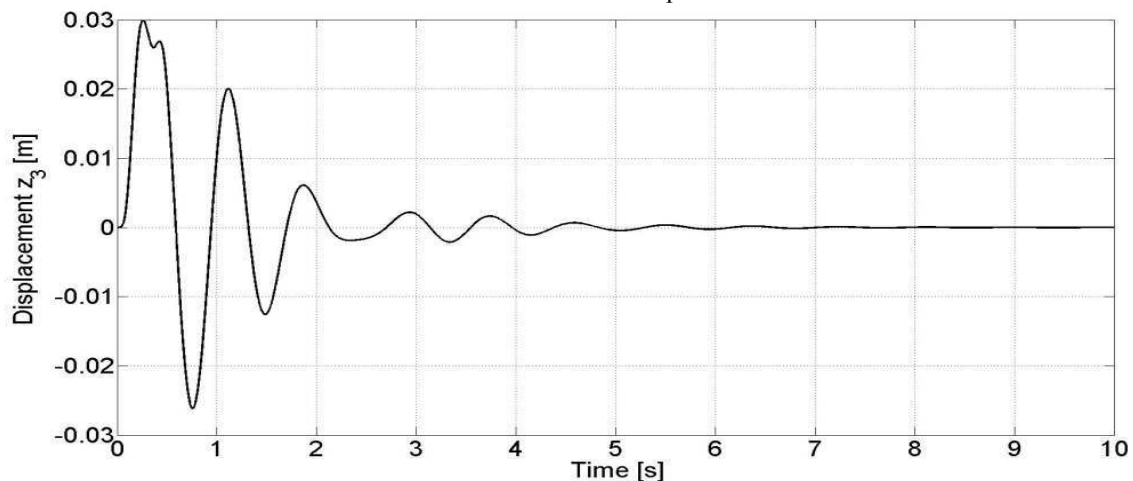


Fig. 6 Displacement of the sprung mass centroid during crossing through the „hat“ obstacle

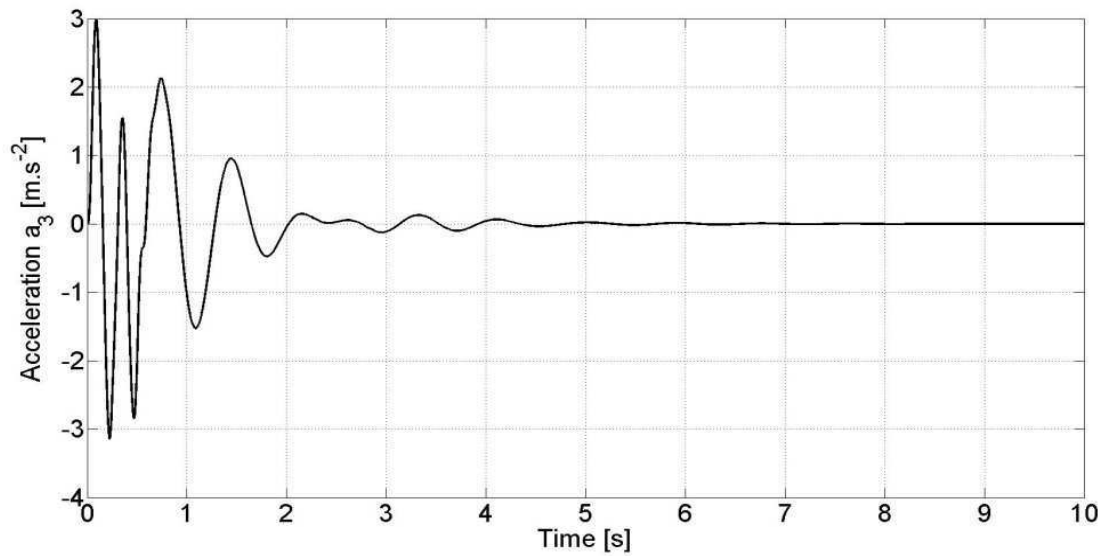


Fig. 7 Acceleration of the sprung mass centroid during crossing through the „hat“ obstacle

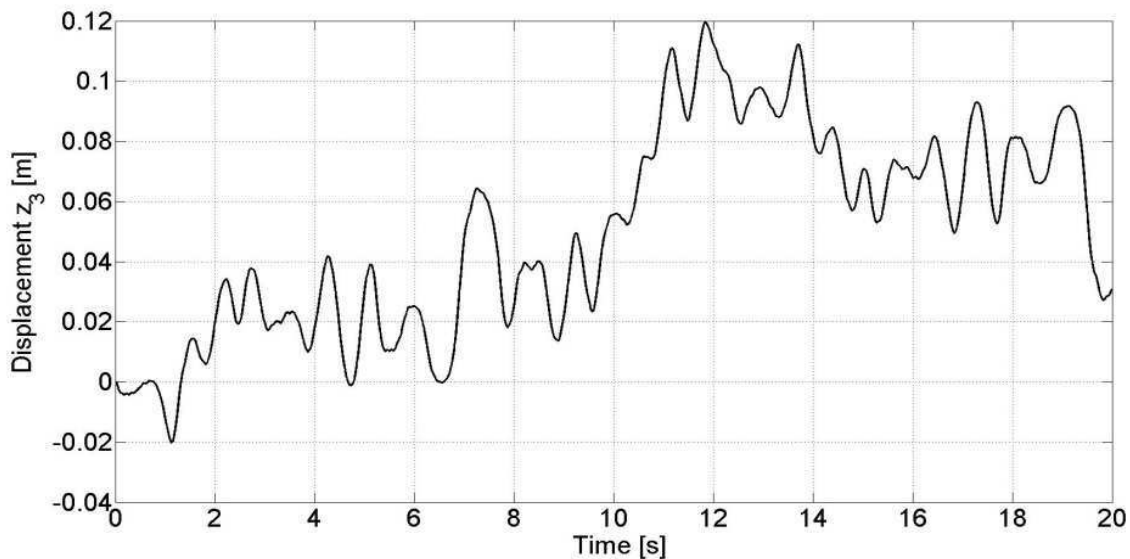


Fig. 8 Displacement of the sprung mass centroid while driving on a stochastically uneven path

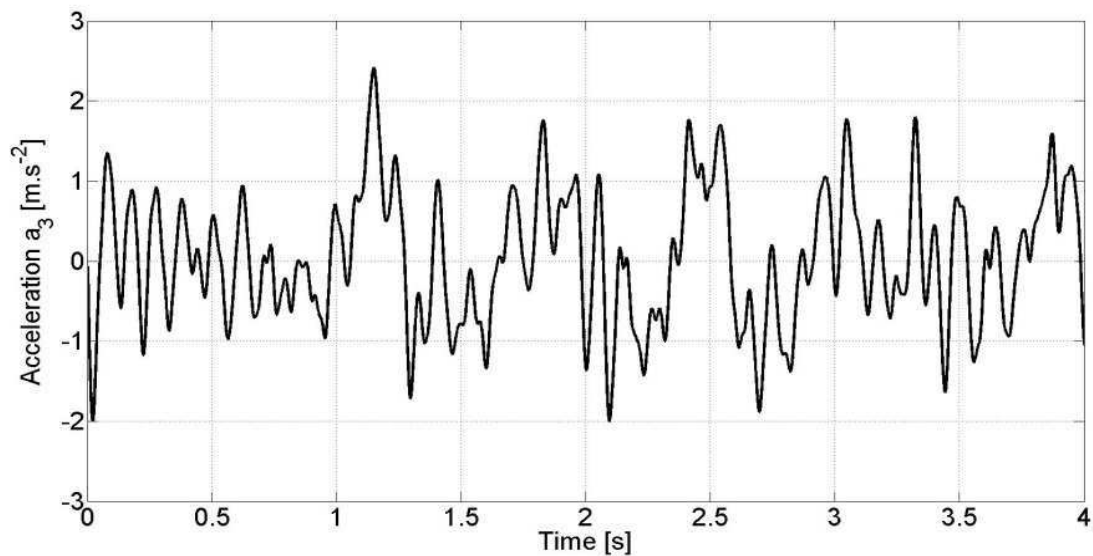


Fig. 9 Acceleration of the sprung mass centroid while driving on a stochastically uneven path

Parameters of the vehicle mechanical model, deterministic obstacles (of cylindrical shape) and vehicle speed when crossing the obstacle are: $m_1 = 110$ kg, $m_2 = 118$ kg, $m_3 = 1370$ kg, $I_3 = 4192$ kg.m², $k_3 = k_4 = 400000$ N/m (tire stiffness), $v = 11$ m.s⁻¹, $h_m = 6$ cm, $d = 2,68$ m. When driving on a rough road stochastic parameters were used: $S_u(\Omega_0) = 22 \cdot 10^{-6}$ m³ and vehicle speed $v = 20$ m.s⁻¹. Parameters of spring stiffness and shock absorber coefficients in the main suspension are: $k_1 = k_2 = 35000$ N/m, $b_1 = b_2 = 1050$ kg.m.s⁻¹.

5 Conclusion

The contribution analyzes and mathematically describes various types of kinematic excitation of vehicles caused by either deterministic obstacles or a stochastically uneven path. It is shown which deterministic obstacles are suitable for simulation testing of transient events and also what obstacles are used on test circuits of prototype vehicles. It also describes a hat-shaped obstacle which, to a certain extent, respects the actual shape of the tire, although it can also be used for the point-to-point contact between the tire and the road.

Planar vehicle model examines its passage over a hat-shaped obstacle and its ride along a stochastically uneven path.

Acknowledgement

This work was supported by the grant project VEGA No. 1/0731/16.

References

- [1] EU 85/3/EW 6 – III: 1985.
- [2] MÚČKA, P. (1999). Odzva modelu nákladného vozidla s aktívnym odpružením na osamelú prekážku. *Strojnícky časopis*, vol. 59, 1999, pp. 265-277.
- [3] NIGAM, N. C., NARAYANAN, S. (1994). *Applications of Random Vibration*. Springer-Verlag, Berlin, 1994.
- [4] Optimization Toolbox for Use with Matlab, User's Guide Version 2 (2000). The MathWorks, Inc., Natick (USA), 2000.
- [5] RAHMAN, M. (1991). *Applied Differential Equations for Scientists and Engineers, Vol. 1. Ordinary Differential Equations*. Computational Mechanics Publications, Southampton, 1994.
- [6] STN 30 0560: 1970.
- [7] VOLEK, J., SEGLA, Š., SOUKUP, J. (2007). *Analytický výpočet vertikálnych posuvů trolejbusu Škoda 21Tr při přejezdu soustavy překážek dle ČSN ve stanovených bodech*. Výzkumná zpráva, Fakulta výrobních technologií a managementu UJEP, Ústí nad Labem, 2007.
- [8] HAUSER, V., NOZHENKO, O. S., KRAVCHENKO, K. O., LOULOVÁ, M., GERLICI, J., LACK, T. (2017). Impact of Wheelset Steering and Wheel Profile Geometry to the Vehicle Behavior when Passing Curved Track, *Manufacturing Technology* ISSN 1213-2489 vol. 17, pp 306-312
- [9] CHALUPA, M., VEVERKA, J. (2016). Handling Simulation of Vehicles, *Manufacturing Technology* ISSN 1213-2489 vol. 16, pp 1264-1269
- [10] KLIMENDA, F., SVOBODA, M., RYCHLIKOVÁ, L., PETRENKO, A. (2015). Investigation of Vertical Vibration of a Vehicle Voted Driving Through a Horizontal Curve, *Manufacturing Technology* ISSN 1213-2489 vol. 15, pp 143-148
- [11] CHALUPA, M. (2015). The Using of Vehicle Moving Simulation to Proposal of Construction Work, *Manufacturing Technology*, ISSN 1213-2489 vol. 15, pp 845-850

DOI: 10.21062/ujep/64.2018/a/1213-2489/MT/18/1/124

Copyright © 2018. Published by Manufacturing Technology. All rights reserved.