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Modeling the Energy Action of vibration and centrifugal forces on the Working Medium and Parts in a Vibration Machine Oscillating Reservoir with an Impeller

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In this paper, the dependences of the tangential component of the velocity of movement of the medium granules, inside the oscillating reservoir, on its radius and oscillation period are obtained. For the analysis, the circulatory motion of working medium granules under the influence of a rotating impeller and the dynamics of a pseudo-gas from abrasive granules exposed to rotating processed parts and an impeller, are considered. From the results, the comparison of the energy impact on the working medium of the rotating processed parts and the impeller is carried out and the distribution of the pseudo-gas velocity from the abrasive granules and its pressure on the surface of the processed parts are obtained. Furthermore, the mechanism of pseudo-gas flow around a rotating part from granules of the working medium is presented. Finally, the schemes of the arrangement of the part in a cylindrical reservoir and its flow around the lateral surface of the rotating part are shown.

Keywords: vibrational treatment, multi-energy technology, superfinishing, free abrasives processing, reservoir with impeller.

1 Introduction

Superfinishing processes are gaining a lot of attention in contemporary industry, as more high quality surfaces are required in mechanical parts [1]. Among the superfinishing processes that are in use, but only a few works can be found in the relative literature, is the proces, where free abrasives are contained within a vibrating reservoir [2, 3]. In the past, for advanced manufacturing processes, modeling with different methods was emplyed in order to obtain a better understanding of the proces and its optimization [4, 5]. The category of free vibrating abrasives was also analyzed through modeling and works while using Finite Elements and Discrete Elements methods can be found in the relevant literature [6, 7].

This work is devoted to the mathematical modeling of the vibration treatment process using an additional force impact on the working medium, including the initiation of the rotational motion of the abrasive granules by an impeller installed on the bottom of the reservoir. The aim of the work is to analyze the mechanism of influence of both oscillatory and circular motion of the working medium and parts on the process. The study is based on the hypothesis that the behavior of the mass of granules under the influence of vibration is similar to that of a gas or liquid. Such a model is actively used to describe a fluidized bed [8-10]. In contrast to the case of a fluidized bed, when a gaseous medium is obtained as a result of blowing through a layer of bulk material, pseudo-gas from abrasive granules arises under the influence of vibrations of the walls of the reservoir and the part [11-14]. Oscillatory motion affects the abrasive granules so that the granules acquire significant kinetic energy and begin to perform movements in the entire volume of the reservoir, similar to the motion of atoms or gas molecules.

Modeling of the process is carried out using the example of the propagation of force action in pseudo-gas from abrasive granules from an external vibrating endless cylinder into it. In addition, it is assumed that there is a circular motion of pseudo-gas from abrasive granules in the inner region of the cylinder. The angular velocity vector of each elementary volume of granules is directed along the axis of rotation of the cylinders. The processed parts, in the form of cylinders with a height approximately equal to their radius, rotate in a pseudo-gas from abrasive granules and are subjected to both vibrations from the walls of the reservoir and the circular motion of the granules.

2 General approach

For the implementation of the multi-energy technology of vibration finishing and grinding processing based on the combined energy action of a complex horizontal vibration and centrifugal nature from the side of the reservoir and the device with the processed parts, a vibrating machine has been created. Its schematic diagram is shown in Fig. 1. The technology and equipment are preferable for operations on removal of burrs, rounding of sharp edges, as well as grinding with a free abrasive medium in order to reduce the surface roughness to Ra=0.32-0.63 μm on box-shaped blanks of parts of the bodies of hydraulic – pneumatic systems after the previous operations of milling and drilling.

Energy sources of complex vibration and centrifugal action on the working medium and processed parts are an impeller and a vertical vibration exciter, a spindle and a vertical vibration exciter from the side of the device for basing and fixing parts.

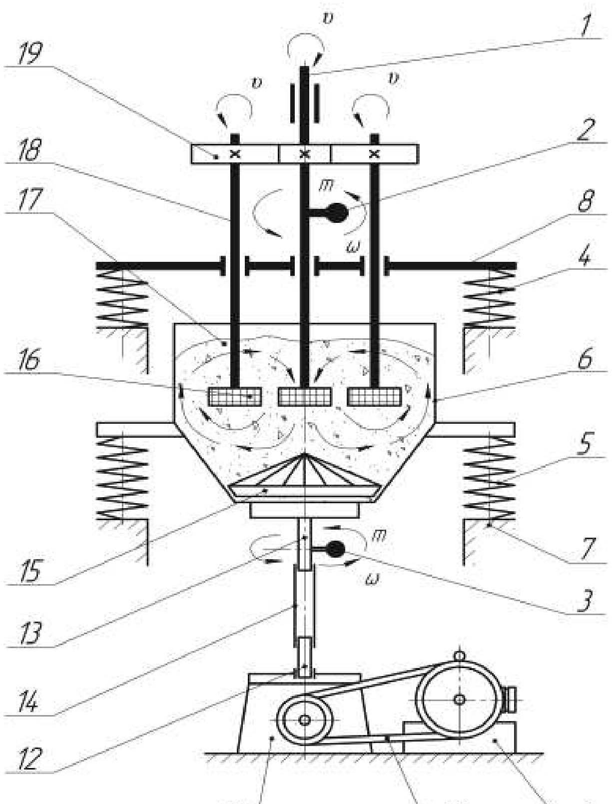


Fig. 1 *Schematic diagram and general view of the vibrating machine arrangement for the implementation of multi-energy technologies: 1 – spindle; 2, 3 – vibration exciter; 4, 5 – elastic suspension; 6 – reservoir; 7 – rigid support; 8 – device; 9 – electric motor; 10 – V-belt transmission; 11 – reducer; 12, 13 – shaft; 14 – flexible coupling; 15 – impeller; 16 – processed parts; 17 – working medium; 18 – locating pins; 19 – gear transmission*

Processing is carried out with the simultaneous use of the energy of centrifugal and vibration forces. Grinding powders, as well as abrasive and metal granules up to 3 mm in size, moistened with a chemically active acid-based solution are used as a working medium. The processing is carried out in an elastically mounted reservoir, which has the shape of hollow figures of a cylinder and a truncated cone with a larger vertical axis, aligned along the conditional plane of the bases. A rotating truncated-conical impeller with a corrugated surface is installed with a large base to the bottom of the reservoir. The impeller shaft is rigidly connected with an inertial vibration exciter, and then through a flexible coupling with the shaft of the bevel gearbox and using a V-belt transmission with an electric motor. In this case, the axis of the vibration exciter shaft located close to the lower outer part of the reservoir coincides with the vertical axis of the reservoir and is perpendicular to its section in the plane of oscillations.

The parts to be processed are based and fixed on the adjusting pins of the device connected to the spindle of the vibrating machine by the gear transmission of the two-pair gearing of the cylindrical wheels. When immersed in the working zone of the reservoir, the device with the processed parts has the ability to simultaneously rotate and oscillate. These movements are excited in the horizontal plane by an inertial vibration exciter located on the vibration machine spindle. The working medium is imparted with a rotational motion at a speed of 50 to 1440 rpm, as well as an oscillatory motion with an amplitude of 0.2 to 3.0 mm and a frequency of 30 to 50 Hz. The spindle of the vibrating machine and the device, rigidly connected to it, with the parts being processed, are submitted to rotational motion at a speed of 31.5 to 1400 rpm, as well as oscillating motion with an amplitude of 0.2 to 3.0 mm and a frequency of 30 to 50 Hz. The spindle and the device with the processed parts are immersed in a reservoir with a circulating working medium. This provides micro-cutting and elastoplastic deformation of the vibration processing. Process control and expansion of its technological capabilities are carried out by using rational combinations of the values of the speeds of rotation of the impeller and the spindle, as well as the values of the amplitude-frequency parameters of their oscillatory motion.

3 Dynamics of the abrasive medium granules under the action of oscillating walls of the vibrating machine reservoir

In the work of Fedorovich and Mitsyk [15], solutions were found for oscillating walls of a cylinder-shape reservoir due to the method of separable variables [16]. In this case, three types of equations were obtained. The equations are the same for both velocity components, which determined the radial H(r), the tangential g(φ), and the time α(t) components of the velocities V_r and V_φ of the elementary pseudo-gas volume from the abrasive granules:

$$x^2\frac{d^2H}{dx^2}+x\frac{dH}{dx}+H(x^2-l)=\frac{lc}{\rho k};\ \frac{1}{g}\frac{d^2g}{d\varphi^2}=l;$$
$$a=Ae^{\pm kt}$$

Constants $k=i\omega$, $l=-1$ are determined from the boundary conditions. A variable x entered during transformations can have two values, either $w=\sqrt{\frac{k}{v}}r$ or $z=i\sqrt{\frac{k}{v}}r$, depending on the sign before the constant k in the expression for $a(t)$. The equation for $H(r)$ is a non-uniform differential Bessel equation, the solution of which has the form [17]:

$$H(x)=\frac{\pi}{2}Y_v(x)\int xJ_v(x)mdx-\frac{\pi}{2}J_v(x)\int xY_v(x)mdx. \quad (1)$$

The solution of equation (1) is determined by the following system of equations:

$$\begin{cases}H(z)=-\frac{\pi}{2}\cdot\frac{c_1c_2c_3}{\rho\omega}\{H_1^*(z)+iH_2^*(z)\};\\ H(w)=\frac{\pi}{2}\cdot\frac{c_1c_2c_3}{\rho\omega}\{H_1^*(w)+iH_2^*(w)\}.\end{cases} \quad (2)$$

The final solution with the boundary conditions $V_{0\varphi}=-A\omega\sin(\omega t-\varphi)$, $V_{0r}=-A\omega\cos(\omega t-\varphi)$ has the form:

$$\boldsymbol{V_r(t,r,\varphi)=\left(F_1^J(r^*)(D_r-B_r)+\frac{\pi}{2}R\omega A^2\left(H_1^*(w)-H_1^*(z)\right)\right)\cos(\omega t-\varphi)} \quad (3)$$

$$\boldsymbol{V_\varphi(t,r,\varphi)=-\left(F_2^J(r^*)(D_\varphi+B_\varphi)+\frac{\pi}{2}R\omega A^2\left(H_2^*(w)-H_2^*(z)\right)\right)sin(\omega t-\varphi)} \quad (4)$$

The coefficients B_φ, D_φ, B_r, D_r included in the expressions (3) and (4) are determined as follows:

$$B_\varphi=\frac{A\omega\left(1-2RA+\pi RAH_2^*(Z)\right)}{2F_2^J(R^*)};\ D_\varphi=\frac{A\omega\left(1+2RA-\pi RAH_2^*(W)\right)}{2F_2^J(R^*)} \quad (5)$$

$$B_r=\frac{A\omega\left(1+2RA-\pi RAH_1^*(Z)\right)}{2F_1^J(R^*)};\ D_r=\frac{A\omega\left(-1+2RA-\pi RAH_1^*(W)\right)}{2F_1^J(R^*)} \quad (6)$$

Here $H_1^*(Z)$, $H_1^*(W)$, $H_2^*(Z)$, $H_2^*(W)$, $F_1^J(R^*)$, $F_1^J(R^*)$, $F_2^J(R^*)$ represent the values of these functions at $r=R$, $r^*=r\sqrt{\frac{\omega}{v}}$ (Fig. 2). The behavior of these functions depending on the dimensionless radius r^*, as well as dependences $V_r(t,r,\varphi)$ and $V_\varphi(t,r,\varphi)$ with $\varphi=0$, are shown graphically (Figs. 3 and 4).

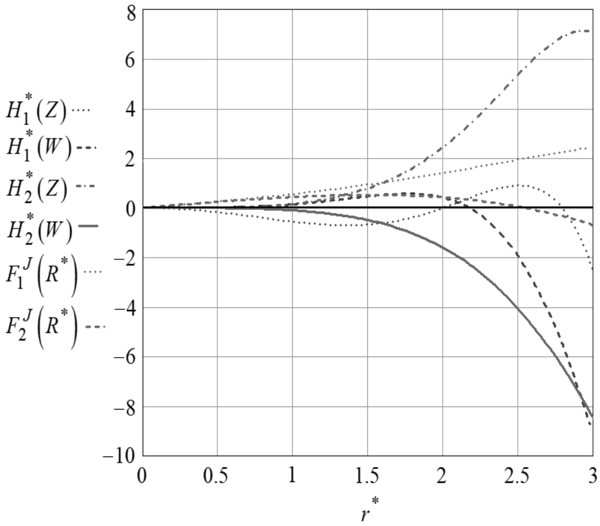


Fig. 2 *Dependence of functions* $H_1^*(Z)$, $H_1^*(W)$, $H_2^*(Z)$, $H_2^*(W)$, $F_1^J(R^*)$, $F_2^J(R^*)$ *on dimensionless radius value* r^*

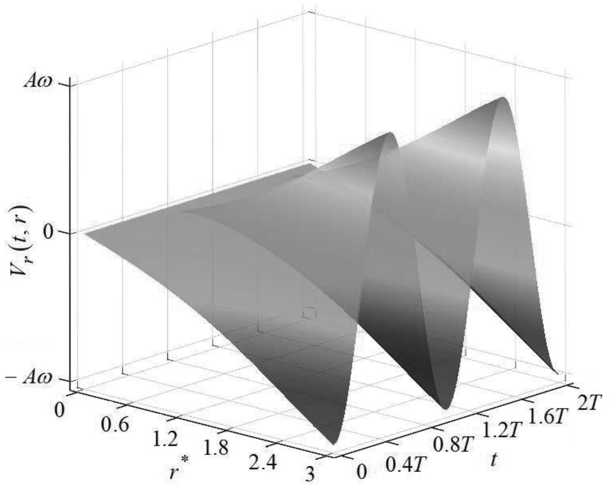


Fig. 3 *Dependence* $V_r(t,\ r,\ \varphi)$ *of the radial component of the velocity of the medium granules inside the oscillating cylinder on its radius and period of oscillation* $T=\frac{2\pi}{\omega}$

Graphic dependence for $V_r(t,r,\varphi)$ shows that the radial $V_r(r^*)$ component of the granule movement velocity is described only along the time axis $t=\frac{2\pi}{\omega}$.

Graphic dependence for $V_\varphi(t,r,\ \varphi)$ describes the oscillatory motion of the working medium, both along the time axis and in the radial direction. Such behavior in the circulation movement of the oscillating working medium is very characteristic of wave processes.

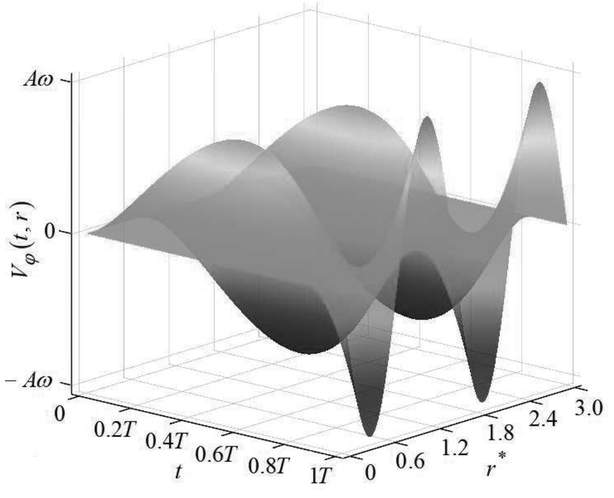


Fig. 4 *Dependence $V_\varphi(t,r,\varphi)$ of the tangential component of the velocity of the medium granules inside the oscillating cylinder on its radius and period of oscillation $T=\frac{2\pi}{\omega}$*

The analogy between the movement of a pseudo-gas from abrasive granules inside an oscillating cylindrical reservoir of a vibration machine and acoustic cylindrical waves confirms that the latter are described by the product of a harmonic function of time and a radial component, which is determined by the Bessel equation [18]. This makes it possible to simulate the process of metal removal during vibration treatment based on the use of multi-energy technology, which consists of the joint action of vibrational and centrifugal forces on the working medium, within the framework of acoustic approximation.

4 Dynamics of pseudo-gas from abrasive granules exposed to rotating processed parts and impeller

In addition to the action of the cylindrical reservoir walls and the impeller, the rotating parts also act on the medium granules. The energy effect of the rotating processed parts on the pseudo-gas can be calculated by determining the friction force of a pseudo-gas formed of abrasive granules on the surface of a rotating part in the shape of a cylinder. The product of this force on the rotation velocity of the part surface is equal to the power transmitted by the part to the abrasive granules.

The dissipation of energy per unit area on the surface of a cylinder of infinite length can be determined from the relation [18]:

$$\frac{\partial e_{surf}}{\partial t}=\left(\vec{v}_\varphi\vec{\sigma}\right), \qquad (7)$$

where expression $\left(\vec{v}_\varphi\vec{\sigma}\right)$ is the scalar product of the velocity vector and the viscous stress vector. Determining the friction force using relation (7) is an approximation from above, since for a cylinder of finite length its velocity will be smaller than in the case of an infinite cylinder due to the fact that the entire infinite surface of a rotating cylinder affects the flow.

Pseudo-gas from abrasive granules is "monatomic" and, according to [19], the stress tensor in such a gas is determined by the formula:

$$\sigma=\frac{4}{3}\nu\mathrm{grad}\vec{v}_\varphi. \qquad (8)$$

In terms of the geometry of our problem and polar coordinates, expression (8) has the form:

$$\sigma=\frac{4}{3}\nu\left(\frac{\partial v_\varphi}{\partial r}-\frac{v_\varphi}{r}\right). \qquad (9)$$

In formulas (8) and (9) as in all the above formulas, the dynamic viscosity is indicated by ν. Thus, expression (7) can be rewritten in the form:

$$\frac{\partial e_{surf}}{\partial t}=\frac{4}{3}v_\varphi\nu\left(\frac{\partial v_\varphi}{\partial r}-\frac{v_\varphi}{r}\right) \qquad (10)$$

Taking into account expression (10), the expression for the power transmitted by the rotating part to the pseudo-gas from abrasive granules is found as:

$$\frac{\partial E_d}{\partial t}=-\int\frac{4}{3}v_\varphi\nu\left(\frac{\partial v_\varphi}{\partial r}-\frac{v_\varphi}{r}\right)ds \qquad (11)$$

Here the integral is taken over the cylindrical surface of the processed part. The minus sign in front of the second term means that the velocity gradient on the surface of the rotating part is negative. In other words, the rotation velocity of the abrasive granules decreases with distance from the surface of the rotating part. The expression for the velocity of movement caused by the rotation of the processed part can be determined as:

$$\frac{\partial^2 Vd_\varphi}{\partial r^2}+\frac{1}{r}\cdot\frac{\partial Vd_\varphi}{\partial r}-\frac{Vd_\varphi}{r^2}=0 \qquad (12)$$

The solution of equation (12), taking into account the boundary conditions on the surface of a rotating cylindrical part and the fact that at infinity the velocity of the circulating motion of a pseudo-gas must decrease to zero, is the following expression:

$$V_{d_\varphi}=\omega_d\frac{R_d^2}{r}. \qquad (13)$$

In expressions (7–13), V_{d_φ} is the tangential velocity of the circular motion of the pseudo-gas, caused by the rotation of the part, ω_d is the angular velocity of rotation of the processed part, and R_d is the radius of the part.

Substituting formula (13) into expression (11), the relation for the power transmitted by the rotating part to the abrasive granules is obtained:

$$N_d=4\pi\nu R_d^2h_d^2\omega_d^2 \qquad (14)$$

where h_d is the height of the processed part. It is accepted that $h_d\approx R_d$.

The equality of power transmitted by the six rotating processed parts and the impeller to the abrasive granules comes to a ratio of $\left(\frac{\omega_d}{\omega_{imp}}\right)^2=400$, which is far from the conditions realized in practice. So, at the maximum angular velocity of rotation of the processed part $\omega_d = 125.6$ rad/s and the average angular velocity of rotation of the impeller $\omega_{imp} = 20$ rad/s, the ratio $\left(\frac{\omega_d}{\omega_{imp}}\right)^2 \leq 40$. That is, the share of energy in the movement of the abrasive granules of six rotating parts is an order of magnitude smaller than the share of the energy of the impeller. This confirms the assumption that the energy share of the circulating flow of a pseudo-gas from abrasive granules and rotating processed parts can be neglected in comparison with the energy of the impeller.

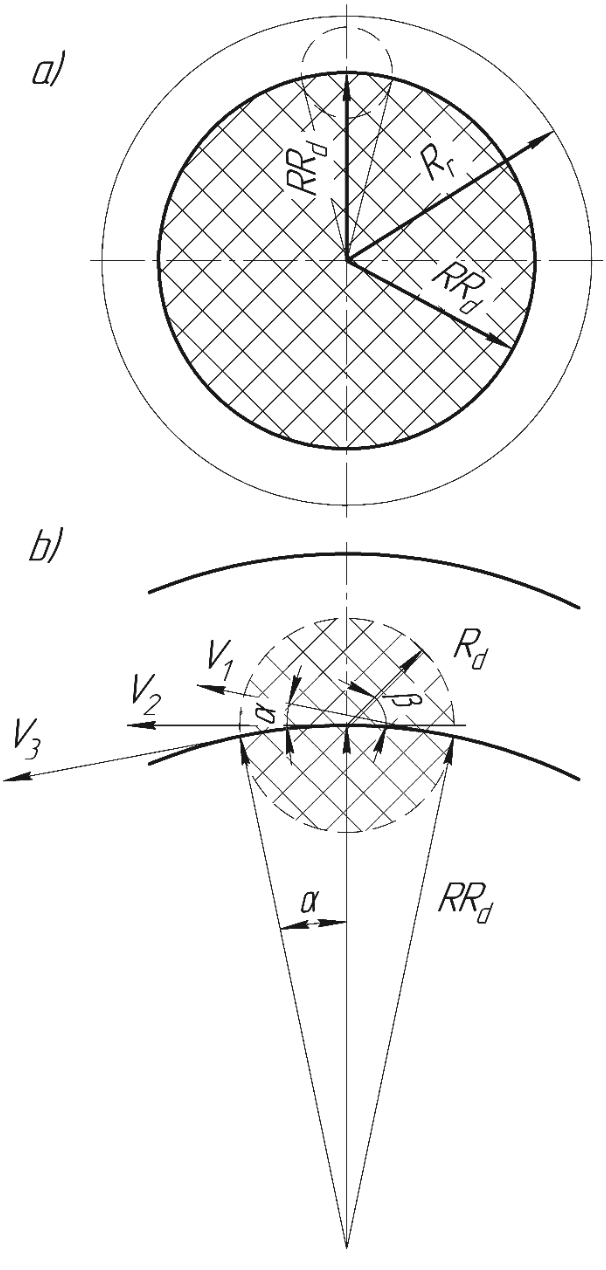


Fig. 5 *Scheme of the location of the part (discontinuous line) in the cylindrical reservoir: (a) the direction of the velocities of the circulation movement of the abrasive granules at a distance from the center of the reservoir of the vibrating machine; (b) the position of the centers of the processed part and the cylindrical reservoir*

Let us consider the process of pseudo-gas flow around rotating parts. In the range of radii 150–200 mm, the velocity of the pseudo-gas flow varies slightly. From the design of the vibrating machine, it follows that the centers of rotation of the parts being machined are in the middle of the interval indicated above, and the diameter of the parts is almost equal to $D_d = 600$ mm. Thus, the interaction of a rotating cylindrical part with a pseudo-gas circulating flow can be reduced to the problem of flow around a rotating cylinder by an incoming rectilinear flow.

Figure 5 shows the following: RR_d is the distance from the center of the reservoir of the vibrating machine to the center of the part; V_1, V_2, V_3 are velocity vectors of the circulation flow of abrasive granules at a distance of RR_d from the center of the reservoir at points corresponding to the intersection of a circle of radius RR_d with the part's border and its middle; α is the angle between the radius vectors passing through the surface of the part of radius R_d and its center; β is the angle between the radius vector RR_d and the tangent to a circle of radius R_d at point A.

Angle α is determined from the ratio $\alpha = \arcsin\left(\frac{R_d}{RR_d}\right)$ and is equal to$9.87°$. The projections of V_1 and V_3 to the direction given by the vector V_2 will be $V_{proj} = |V_1|\cos\alpha = |V_3|\cos\alpha = |V_1|0.985$ (Fig. 5, b). Thus, V_{proj} differs from V_1 or V_2 by 1.5 % and it can be considered that the rotating processed part is flowed around by the flow of pseudo-gas from abrasive granules with the velocity of the abrasive granules circulating flow, which is realized at a distance of RR_d from the center of the vibrating reservoir.

The solution of the problem of flow around a rotating cylinder is quite common [20]. The relations for the radial and tangential components of the flow velocity are well known:

$$\begin{cases} V_{dr} = V_\varphi\left(1 - \frac{R_d^2}{r_d^2}\right)\cos\beta; \\ V_{d\beta} = -V_\varphi\left(1 + \frac{R_d^2}{r_d^2}\right)\sin\beta + \frac{\Gamma}{2\pi r_d} \end{cases} \quad (15)$$

here V_{dr} and $V_{d\beta}$ are the radial and tangential components of the velocity, and Γ is the velocity circulation over the surface of the part. Furthermore,

$$\Gamma = 2\pi\ R_d^2\omega_d, \quad (16)$$

where ω_d is the angular velocity of rotation of the part, R_d is the radius of the rotating part.

As can be seen from the first relation of system (15), the radial velocity on the surface of the part is zero.

Since the motion in the reservoir is steady, to determine the pressure on the surface of the rotating processed part the Bernoulli's theorem for the liquid flow can be used:

$$P_\infty + \rho\frac{V_\infty^2}{2} = P_d + \rho\frac{V_{d\beta}^2}{2}, \quad (17)$$

where P_∞ and V_∞ are the pressure and velocity of the unperturbed flow $(V_\infty = V_\varphi)$ and P_d and $V_{d\beta}$ are pressure and tangential velocity on the surface of the part.

Equation (17) describes the ratio of pressures and velocities, without disclosing the reasons for their appearance. However, in this case, as in others, it is necessary to determine the forces that are the cause of the flow field. It has been shown in [21, 22] that in the presence of conservative forces $(F_{cons} = \text{grad}U)$ the equation (17) should be supplemented with the term describing the potential of these forces – U:

$$P_\infty + \rho\frac{V_\infty^2}{2} + U_\infty = P_d + \rho\frac{V_{d\beta}^2}{2} + U_d \quad (18)$$

In case examined in this paper, the field of forces acting on the abrasive granules is not conservative, but these forces must be taken into account before using the Bernoulli's theorem. When finding the pressure distribution over the surface of the rotating processed part located in the pseudo-gas stream of abrasive granules, some factors should be considered. The force of the impeller action, which is replaced by the force distributed over the reservoir radius per unit volume:

$$|F(r)| = 8v\omega_{imp}\frac{R_{imp}^2}{HR_r^3}\sqrt{\frac{kC_{imp}}{2}}r \quad (19)$$

the pressure caused by the rotation of the part being processed:

$$P_{\text{rot}d} = \rho_{pc}\frac{\omega_d^2 R_d^4}{2r^2}; \quad (20)$$

and the force acting in the direction tangential to the radius of the part, caused by friction (by viscosity) between the surface of the rotating part and the pseudo-gas from abrasive granules:

$$F_{frd} = 2v\pi R_d h_d\left(\frac{\partial V_{d\beta}}{\partial r} - \frac{V_{d\beta}}{r}\right) \quad (21)$$

Some remarks on the influence of each of these forces should be made. Imagine a streamline flowing around the side surface of a rotating processed part (Fig. 6). The rotation of the part corresponds to a positive value of the circulation Γ.

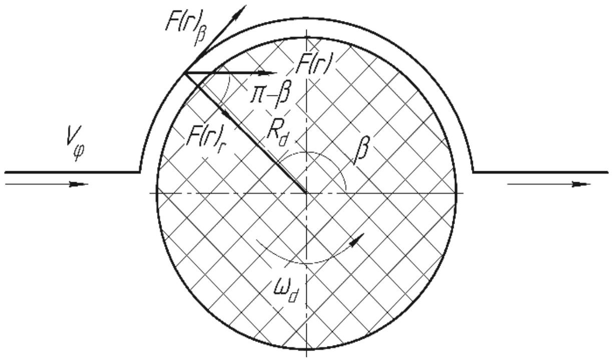


Fig. 6 *Scheme of the flow around a rotating processed part curved lateral surface by a liquid flow*

It is obvious that the force $F(r)$, which determines the impeller action on the abrasive granules, can be decomposed into two components – radial $F(r)_r$ and tangential $F(r)_\beta$ (Fig. 6). The radial component acts on the surface of the part, creating additional pressure that must be taken into account when calculating the removal of metal. The tangential component creates an additional pressure that acts on the pseudo-gas flow flowing around the side surface of the rotating part. The expression for the pressure that is created by the force $F(r)$ acting on the unit volume of the pseudo-gas can be found from the relationship:

$$P_{F_{imp}} = F(r)2\pi r h_d\frac{\Delta r}{S_{norm}} = F(r)2\pi r h_d\frac{\Delta r}{\Delta r h_d} = 2\pi F(r)r \quad (22)$$

where S_{norm} is the area, perpendicular to the velocity of the circulation flow of the pseudo-gas created by the rotation of the impeller, in the form of a "ring" of radius r, with a width of Δr and a height of h_d.

If, for simplicity, it is assumed that force $F(r)$ is constant to the diameter of the part along the entire transverse circulation flow and is equal to $F(RR_d)$, then the expression of pressure created by the force of rotation of the impeller can be written as:

$$P(RR_d)_{F_{imp}} = 4\pi^2 8v\omega_{imp}\frac{R_{imp}^2}{HR_r^3}\sqrt{\frac{kC_{imp}}{2}}RR_d^2 \quad (23)$$

here the radius at which the centers of the parts are located $RR_d = 175$ mm.

Let us determine the pressure created by the force acting in the direction perpendicular to the radius of the part, caused by friction (by viscosity) between the surface of the rotating part and the pseudo-gas of the working medium granules. Substituting relation (13) into expression (21), it is obtained:

$$F_{frd} = -4v\pi R_d^3 h_d\frac{\omega_d}{r^2}. \quad (24)$$

A diagram for the derivation of the expression of pressure generated by force F_{frd} is shown in Fig. 7. From the scheme it can be seen that the pressure in the thin layer Δr near the surface of the rotating part will be equal to:

$$P_{frd} = \frac{\left(F_{frd}(R+\Delta r) + F_{frd}(R)\right)}{2\Delta r h_d} \quad (25)$$

Turning to infinitesimal values of r:

$$P_{frd} = \frac{1}{2h_d}\cdot\frac{dF_{frd}}{dr} \quad (26)$$

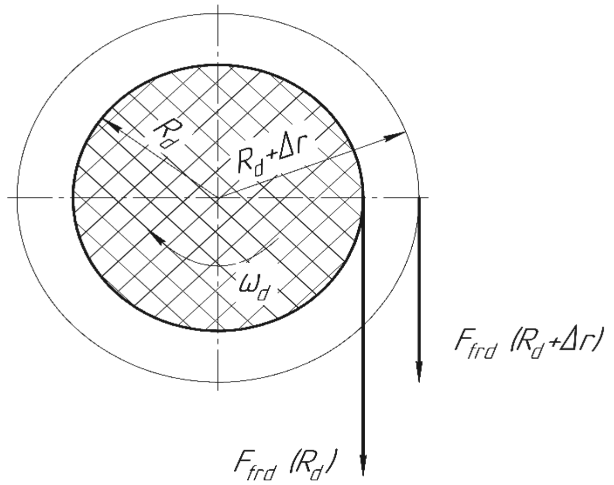


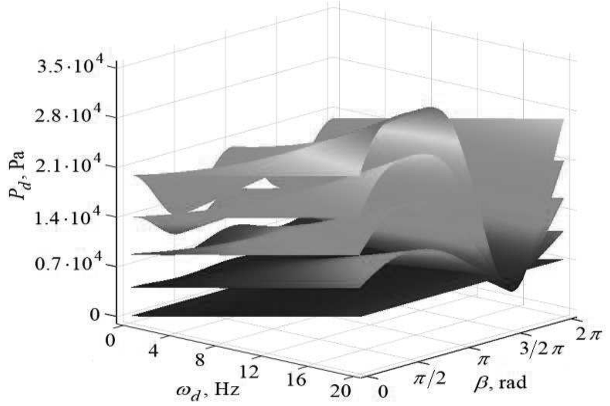
Fig. 7. *Diagram of the derivation of the expression of pressure created by force* F_{frd}

Substituting expression (24) into equation (26), the final expression of the pressure on the part created by the force acting in the direction perpendicular to the radius of the part, and caused by friction (by viscosity) between the surface of the rotating part and the pseudo-gas from the abrasive granules is obtained:

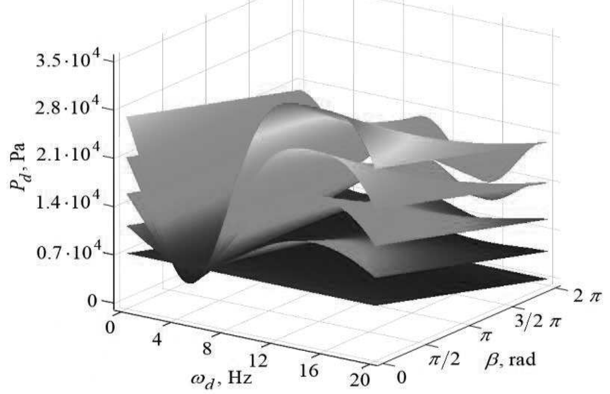
$$P_{frd} = v2\pi\omega_d \tag{27}$$

Equation (18) can be considered as the law of conservation of energy of a single volume of liquid or gas. In it, the pressure plays the role of the potential energy of the gas, the expression for the pressure $\rho \frac{V^2}{2}$ plays the role of kinetic energy, and U is the potential energy of external forces. The field of external forces is not conservative, that is, it cannot be represented by the function of the potential U. Therefore, the influence of external forces through the pressure created by them is expressed. Based on the above, it is necessary in equation (18) to substitute expressions (20), (23) and (27) for the potential U_∞. In this case, it is necessary for U_d be equal to zero. The final expression of pressure on the surface of the rotating processed part will be in the form:

$$P_d = P_{imp} + \rho\frac{V_\varphi^2}{2} + P_{\text{rot}d} + P(RR_d)_{G_{imp}} + P_{frd} - \rho\frac{V_{d\beta}^2}{2} \tag{28}$$



a)



b)

Fig. 8 *Plots of pressure* P_d *versus angle* β *and the angular velocity of rotation of the part for various values of the impeller angular velocity. The graphical dependencies in Fig. 8 (a) correspond to the rotation of the processed part and the impeller in one direction, while the graphs in Fig. 8 (b) correspond to the opposite direction of rotation of the processed part and the impeller*

For calculations using formula (28), it is necessary to use relations (15), (20), (22), (23) and (27). The full expression for pressure P_d is not given here because of its bulkiness. The graphs of pressure P_d versus angle β and the angular velocity of rotation of the part for various values of the impeller angular velocity are shown in Fig. 8.

5 Conclusion

From the preceding analysis some conclusions may arise:

The use of multi-energy technology, which consists in the combined action of vibration and centrifugal forces on the working medium, increases metal removal during vibration treatment by a factor of 1.6 to 1.8. This result was obtained experimentally when processing steel box-shaped body parts of electrical equipment. The metal removal was weighed, before and after processing with an accuracy of 0.001 g, equal to the achieved surface roughness. The results of the experiments confirmed the previously given theoretical premises.

- In this case, the share of metal removal from the action of vibrating walls of the reservoir in the total metal removal decreases when the rotation speeds of the processed part and the impeller increase. However, it is precisely the oscillations of the reservoir wall that create a mobile circulating medium of abrasive granules, which makes it possible to process the surface of the part in conditions of optimizing the magnitude of the energetic effects of the rotating parts and the impeller.

- The use of a rotating impeller and part rotation, combined with the action of vibrating reservoir walls, create conditions under which different sections of the part surface are processed in different ways. The selection of modes of multi-energy action allows to regulate the vibration treatment process in the desired direction (micro-cutting, elastic-plastic deformation, etc.), as well as to combine these processes in certain combinations.
- It has been established that the processing of parts with granules of the working medium according to multi-energy technology occurs due to the oscillatory actions of the working surfaces of the reservoir and the centrifugal effects of the corrugated impeller, as well as due to the own rotation of the parts around its axis.

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