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Optimization of Tooth Profile Modification and Backlash Analysis of Multi-tooth Mesh Cycloid Transmission

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Cycloid gear drive is widely used in robot cycloid planetary reducer, and the transmission accuracy is the key property of the reducer. The standard cycloid transmission is a multi-tooth mesh. The modification method has attracted extensive attention as one of the important parameters of the cycloid drive. The influence of the isometric and shifting modification of the cycloid gear on the cycloid transmission backlash was analysed according to the characteristics of the multi-tooth mesh and the profile equation of the modified cycloid gear in this study. Combined with the backlash analysis, a multi-objective optimization mathematical model of cycloid gear modification parameters was established to ensure the backlash and strength of the reducer. The study showed that the modification combination mode and parameters were obtained under different application conditions, thus providing a certain reference for the modification parameter design of cycloid transmission.

Keywords: Cycloid transmission, Tooth profile, Modification, Backlash, Optimization

1 Introduction

A cycloid planetary reducer with cycloid gear drive is widely used in robots because of its high precision, high rigidity, and high reliability [1-3]. Tooth modification is usually necessary to reshape the cycloid gear so as to ensure the accuracy and meet the requirements of lubrication, assembly, and service life. In the standard cycloid drive, ideally, half of the teeth on a single cycloid gear are meshed with the needle teeth simultaneously, and the clearance of all the mesh tooth pairs is zero [4-6]. In practical application, it is necessary to modify the standard tooth profile of the cycloid gear so as to compensate manufacturing error, temperature fluctuation, and loading deformation of the tooth surface; facilitate assembly; and ensure lubrication of mesh pair[7,8]. The backlash of the cycloid reducer is one of the most important precision parameters in industrial robot applications that need precise positioning during reciprocating motion. The tooth shape of the cycloid gear is corrected to produce a mesh backlash.Therefore, the modification has become an important factor affecting the performance of the cycloid planetary reducer, such as backlash, lubrication, and service life [9-13].

Some studies have been performed on the profile and modification of cycloid gears, such as the study by Weidong et al. [14], who studied and put forward a modification scheme mainly using the combinations of three methods. The methods included equidistant modifications, rotation angle modification, and radial-moving modification. Many studies put forward various modification methods. The basic starting point was to improve the performance of the reducer according to the load condition of the cycloid gear drive and meet lubrication and assembly requirements. For example, Tianmin [15] put forward the reverse bow profile of a new-style three-disk-cycloid-driving, which had three pieces of cycloidal gear evenly distributed at 120° in the eccentric direction, considering the best bearing tooth surface engagement state, and adopted the bow back profile to improve the accuracy in situations with high accuracy requirements. With the development of cycloid grinding technology[16, 17], various tooth profiles can be processed. The corresponding studies on tooth modification put forward various modification modes, such as parabola modification and pressure angle modification. Their idea was basically to use different modification modes for the participation of the tooth profile in the conjugate mesh segment, approaching the theoretical profile to guarantee the accuracy of cycloid transmission, while the nonparticipating mesh segment was modified to meet the lubrication and assembly needs [18, 19].

According to the principle of the cycloid transmission, this study analyzed the influence of the equidistance and radial-moving modification on the backlash in the multi-tooth mesh cycloid transmission gear taking tooth modification into account. Combined with the backlash analysis, a multi-object mathematical model for the optimization of the cycloid gear tooth profile was established. Also, the modification parameters to ensure the accuracy and strength were obtained through the calculation example, so as to provide a tool and references for improving the performance of cycloid gear transmission. [22-24]

2 Materials and Methods

2.1 Tooth profile and its modification for cycloid gear

2.1.1 Tooth profile of the cycloid gear

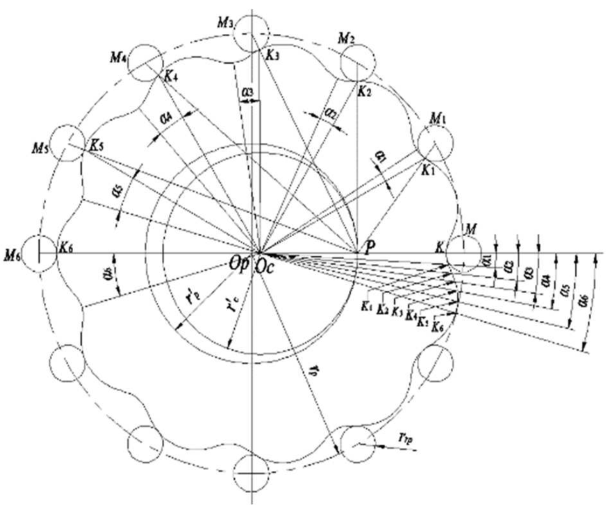


***Fig. 1** Mesh of cycloid gear and needle gear*

Cycloidal gear profiles are formed by external and internal rolling methods. The method of generating epicycloid was the epicycloid is generated by the trajectory of a point on the circumference of a rolling circle when the rolling circle is rolling purely on the circumference of the base circle. Generally, the tooth profile of cycloid drive is the track of a point in the circle, which is called short amplitude epicycloid. Fig. 1 shows the meshing of the cycloid gear with the pin gear [20]. According to the meshing principle of the cycloid drive, two pitch circles are tangent to node P; r'_c and r'_p are the pitch radii of the cycloid and of the pin gear, respectively. The meshing points of the pin gear and the cycloid gear are K, K_1, …, K_6. The centers of the needle teeth are M, M_1, …, M_6. O_c and O_p are the geometric centers of the cycloid gear and of the needle wheel, respectively.

2.1.2 The cycloid gear tooth modification

Theoretically, in the standard cycloid drive, the cycloid gear meshes with a pinion without side clearance. It cannot be applied directly to the drive because it cannot compensate the manufacturing and assembly errors. Therefore, the tooth profile of the cycloid gear needs to be modified in actual design and manufacture. On the one hand, the clearance formed by the cycloid gear tooth modification can provide compensation for manufacturing and assembly errors. On the other hand, it can provide clearance for gear lubrication in the parts of the tooth top and tooth root, which are not implicated in the meshing.

Based on the generating tooth grinding principle of cycloid tooth, the corresponding tooth modification methods, including rotation angle modification, radial-moving modification, and equidistant modification, were generated, as shown in Fig. 2 [21, 25]. Among these, the rotation angle modification referred to the second grinding cycloid gear by rotation angle after the standard profile was ground. The tooth thickness of the profile with rotation angle modification was reduced to form a clearance so as to supplement the manufacturing error and meet the lubrication requirements. The utility model had the advantages that the conjugate tooth shape was still meshed and the motion transmission was continuous after the modification. However, the rotation angle modification could not produce radial clearance at the top and root of the tooth, and therefore it was not used alone. Equidistant profile modification was achieved by adjusting the radius of the grinding wheel. Its essence was to shift the standard profile along the normal direction by the radius variation in the grinding wheel. The distance between the grinding wheel and the workpiece was adjusted to achieve the profile radial-moving modification. The offset at the top and the root of the tooth profile was the adjusted distance. The offset in the rest section of the tooth profile was less than the adjusted distance and changed with the mesh phase angle.

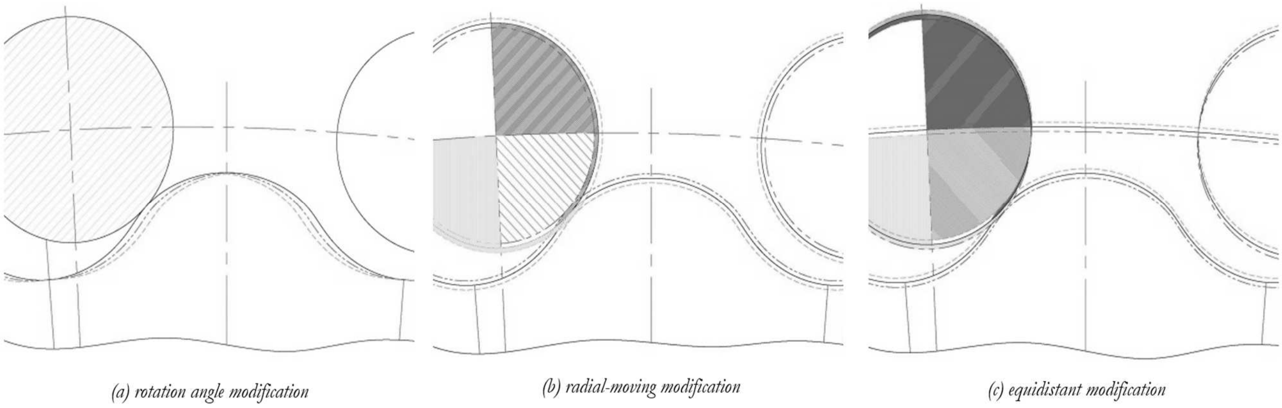


Fig. 2 *Tooth profile with different modification style*

Where:

(a)…Black -standard; red –(+0.2mm); green- (+0.4mm),

(b)…Black - standard; green- (+0.2 mm); red- (-0.2 mm),

(c)…Black - standard; green- (+0.2 mm); red- (- 0.2 mm).

The combination of radial-moving modification and equidistant modification was usually used to achieve the conjugate continuous transmission with rotation angle modification and generate the gap at the top and bottom of the tooth. Through the combined modification, the engagement segment was close to the profile with rotation angle modification, and the mesh gap at the positions of the tooth top and tooth root was produced. This kind of modified tooth profile was approximate to the ideal transmission tooth profile, which did not completely meet the conjugate mesh conditions. Therefore, once the design of tooth profile modification parameters was unreasonable, it was easy to have different problems, such as large backlash error, speed ratio fluctuation, and bearing capacity reduction.

Based on the analysis of the modification method, the following main factors should be considered in the modification parameter design.

- 1) The working part of the tooth profile needs to satisfy the conjugate mesh condition, and the appropriate clearance should be reserved to compensate the manufacturing and assembly errors so as to ensure the continuity, accuracy, and small backlash of the transmission.
- 2) Suitable clearance should be reserved for the no-working section of the tooth top and root to meet lubrication needs.
- 3) The entire tooth profile should be smooth and continuous.

2.1.3 Influence of cycloid gear modification on backlash

Backlash is an important performance parameter of precision cycloid drive in the condition with high reciprocating-motion positioning accuracy requirements. It is the value of the angle at which the output axis lags behind the input axis from the start of the reverse motion of the input axis to the time when the output axis follows the reverse motion.The cycloid tooth modification parameters were designed to produce the gap between the theoretical mesh teeth pairs, which also caused backlash in cycloid transmission, to compensate the errors caused by manufacturing and assembling.

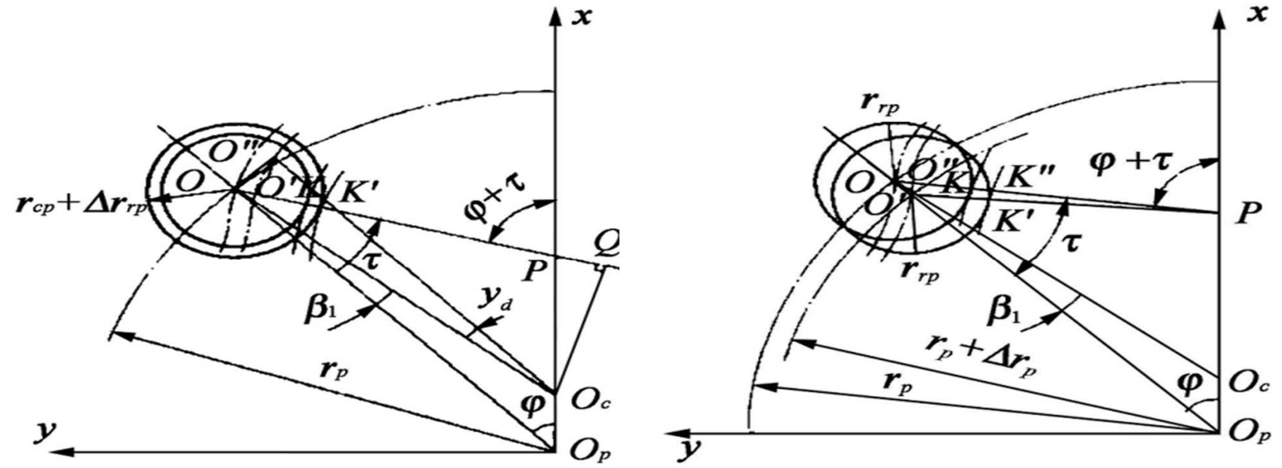
For the correction of the angle of rotation, the angle of rotation of the cycloid gear and the corresponding angle of thinning of the cycloid gear in the circumferential direction are the theoretical backlash caused by the correction of the cycloid gear. When a modification is made to a cycloid gear, the cycloid gear needs to be rotated at an angle to eliminate the backlash between the cycloid gear and the pin gear caused by the modification, which is the theoretical backlash of the cycloid drive caused by the modification. It was necessary to analyze the influence of modification on the backlash to ensure the cycloid transmission backlash and design tooth modification parameters.

2.1.4 Backlash caused by equidistant modification

Fig. 3(a) is an analysis diagram of equidistant modification. When the cycloid gear with equidistant modification was meshed with the standard pinwheel, a gap ($\overline{KK'}$)was found along the direction of the common normal line OP , and ($\overline{KK'}$) was equal to the distance between the point O on the theoretical standard profile and the point O' on the theoretical profile with modification, which was also equal to the size of the equidistant profile modification amount, that is:

$$\overline{KK'} = \overline{OO'} = \Delta r_{rp} \,, \tag{1}$$

It meant that when the cycloid gear was equidistant modified, all the needle teeth and cycloid gear teeth had equal clearancein the direction of the common normal. The cycloid gear should be rotated around its center O_c by the angle γ_d to eliminate the clearance. The center of the theoretical modified tooth profile moved from O'' to the point O. As shown in Fig. 3(a), when the value $\cos\varphi = K_1$, the rotation angle of the cycloid gear γ_d reached the minimum value $\gamma_{d\min}$.



(a) Equidistant modification (b) Radial-moving modification

Fig. 3 *Calculation diagram of backlash caused by modification*

$$\gamma_{d\min} = \frac{\Delta r_{rp}}{a \cdot z_c} \tag{2}$$

Where:
a ...The eccentricity,
z_c …The tooth number of the cycloid gear,
$a \cdot z_c$ …The pitch circle radius of the cycloid wheel.

Therefore, when the entire cycloid gear rotated by an angle $\gamma_{d\min}$, the needle teeth at the position of $\varphi = \varphi_0 = \arccos K_1$ contacted the tooth surface of the cycloid gear. However, still a gap existed between the needle teeth and the cycloid gear tooth surface at other positions. Considering a rotational angle in the positive and negative directions of the cycloid gear, the minimum rotation angle required for making up the backlash caused by equidistant modification of the cycloid gear was $\frac{2 \cdot \Delta r_{rp}}{a \cdot z_c}$.

2.1.5 Backlash caused by radial-moving modification

Fig. 3b is the analysis diagram of radial-moving modification. When machining the cycloid gear, the center of the grinding wheel was O' and the contact point was K', while when the cycloid gear meshed with the standard needle wheel, the center of the needle wheel was on the extension line of O_pO', and the common normal line was OP. Therefore, the clearance at the relative position in the diagram was:

$$\overline{KK''} = \overline{OO''} = \overline{OO'}\cos\tau = -\Delta r_p \cos\tau \tag{3}$$

The cycloid gear should rotate by an angle of γ_Y around its center O_c to eliminate the clearance. A series of formula derivations was used to obtain the result that the rotation angle γ_Y of the cycloid gear was the minimum value $\gamma_{Y\min}$ when $\cos\varphi = K_1$.

$$\gamma_{\mathrm{Ymin}} = \frac{-\Delta r_p}{a z_c}\sqrt{1-K_1^2} \tag{4}$$

Considering that angles existed in both positive and negative directions of the cycloid gear, the minimum angle that the cycloid gear needed to rotate was:

$$\frac{-2\Delta r_p}{a z_c}\sqrt{1-K_1^2} \tag{5}$$

When the pair of teeth with the smallest lost motion angle contacted with the needle teeth, it drove the reducer to reverse; therefore, the backlash caused by the combined modification of equidistant and radial-moving was:

$$\Delta\varphi_1 = \frac{2\Delta r_{rp}}{a \cdot z_c}\frac{-2\Delta r_p}{a \cdot z_c}\sqrt{1-K_1^2} \tag{6}$$

2.1.6 Optimization of cycloid gear profile modification based on multi-tooth mesh and backlash analysis

The multi-tooth mesh of the cycloid drive led to high accuracy and high rigidity, which required reasonable tooth modification parameters to ensure the assembly and transmission efficiency at the same time. Therefore, a multi-objective optimization model for profile correction of cycloidal gears was established based on the backlash analysis, and the parameters of profile correction of cycloidal gears were optimized.

2.1.7 Objective function for optimization

It was necessary to ensure that the working part of the tooth profile was as close to the angle profile as possible to meet the conjugate mesh conditions, and the backlash was as small as possible to meet the transmission accuracy requirements, so as to optimize the cycloidal gear profile. Therefore, the proposed mathematical model had two objective functions.

(1) Objective function 1: Make the tooth profile with combined equidistant and radial-moving modification close to the tooth profile with rotational angle modification, so as to ensure the continuity of transmission.

According to the rotational angle modification amount δ required by the side clearance and the teeth number of simultaneous mesh, the phase angle φ between the two boundary points of the working section part of the cycloid gear tooth profile, which coincided with rotational angle modification tooth profile was preliminarily determined. In this interval, φ was divided into $m-1$ equal parts. Based on the tooth profile formula with rotational angle modification, the tooth profile coordinates $\left(x'_{ci}, y'_{ci}\right)$ $(i=\mathbf{1},\ldots,m)$ were obtained. At the same time, the rotational angle modification $\delta=0$, that is to say, y_{ci} was equal to y'_{ci}, which was in the coordinate $\left(x'_{ci}, y'_{ci}\right)$ of the tooth profile with combined isometric and radial movement retrofit. Then, the phase angle of the isometric and radial movement retrofit was obtained, which was φ'_i, followed by x'_{ci}. The average of the algebraic sum of the absolute value of the difference between the x coordinate corresponding to the combined modification and the rotational angle modification was taken as the objective function to measure the proximity of the two kinds of profile, namely:

$$F_1(\Delta r_{rp}, \Delta r_p) = \frac{1}{m}\sum_{i=1}^{m}\left|x'_{ci} - x_{ci}\right| \tag{7}$$

(2) Objective function 2: Ensure a certain theoretical backlash value: The backlash of the cycloid drive with tooth modification was taken as the second objective function, and the backlash caused by modification was:

$$F_2(\Delta r_{rp}, \Delta r_p) = \frac{2\cdot\Delta r_{rp}}{a\cdot z_c} - \frac{2\cdot\Delta r_{rp}}{a\cdot z_c}\sqrt{1-K_1^2}\,(\mathrm{rad}) \tag{8}$$

2.1.8 Optimization constraints

The initial mesh backlash of the cycloid gear with equidistant and radial-moving modification was:

$$\begin{aligned}\Delta(\varphi) = {} & \Delta r_{rp}\left(1-\sin\varphi S^{-\frac{1}{2}}\right) - \\ & \Delta r_p\left(1-K_1\cos\varphi-\sqrt{1-K_1^2}\sin\varphi\right)S^{-\frac{1}{2}}\end{aligned} \tag{9}$$

At the value of $\varphi = \cos\mathrm{arc}K_1$, $\Delta(\varphi)=0$ was the extreme point.

$$\begin{aligned}\frac{d^2\Delta(\varphi)}{d\varphi^2} = {} & \Delta r_{rp}\left(-3K_1^2S^{-2}\sin^2\varphi+3S^{-1}K_1\cos\varphi+1\right) \\ & S^{-\frac{1}{2}}\sin\varphi-\left(K_1\cos\varphi+\sqrt{1-K_1^2}\sin\varphi\right)S^{-\frac{1}{2}} - \\ & \Delta r_p\left(\sqrt{1-K_1^2}\cos\varphi-K_1\sin\varphi\right)K_1S^{-\frac{3}{2}}\sin\varphi - \\ & \Delta r_p\left(1-K_1\cos\varphi-\sqrt{1-K_1^2}\sin\varphi\right) \\ & \left(3K_1\sin^2\varphi-S\cos\varphi\right)K_1S^{-\frac{5}{2}}\end{aligned} \tag{10}$$

$$\left.\frac{d^2\Delta(\varphi)}{d\varphi^2}\right|\varphi = \arccos K_1 = \Delta r_{rp} - \Delta r_p\sqrt{1-K_1^2} \tag{11}$$

As shown in Equation (9), the side gap produced by equidistant modification was larger than that produced by radial-moving modification for the same size of isometric and radial movement retrofit. The radial clearance at the position of tooth top and tooth root was $\Delta r > 0$, and $\Delta r = \Delta r_{rp} - \Delta r_p$. The specific combination forms of equidistant and radial-moving combined modification were as follows:

(a) Positive equidistance + positive radial-moving. These two modification methods partly offset each other. Moreover, the side gap produced by equidistant modification was larger than that caused by the same size of radial-moving modification. To ensure positive side clearance, the equation should be satisfied so that $\Delta r_{rp} > \Delta r_p$. Therefore, the optimal constraints of the combined modification were as follows.

$$\Delta r_{rp} > \Delta r_p, \Delta r_{rp} > 0, \Delta r_p > 0 \tag{12}$$

(b) Positive equidistance + negative radial-moving. Since the combined modification was superposed in the direction of increasing the side gap, the optimal constraint constraints were as follows:

$$\Delta r_{rp} > 0, \Delta r_p < 0 \tag{13}$$

(c) Negative equidistance + positive radial-moving $\Delta r_{rp} - \Delta r_p < 0$; this combined modification could not produce radial clearance at the region of root and top of tooth, and hence it could not be used.

(d) Negative equidistance + negative radial-moving. The negative equidistance modification resulted in a decrease in the side clearance, and the negative radial movement retrofit resulted in the increase in the side clearance. In combination with Equation (9), the optimal constraint for the combined modification was:

$$\Delta r_{rp} - \Delta r_p\sqrt{1-K_1^2} > 0, \Delta r_{rp} < 0, \Delta r_p < 0 \tag{14}$$

Therefore, a multi-objective optimization model for modifying the parameters of the cycloid pinwheel drive was developed and optimized by programming.

3 Discussion of results

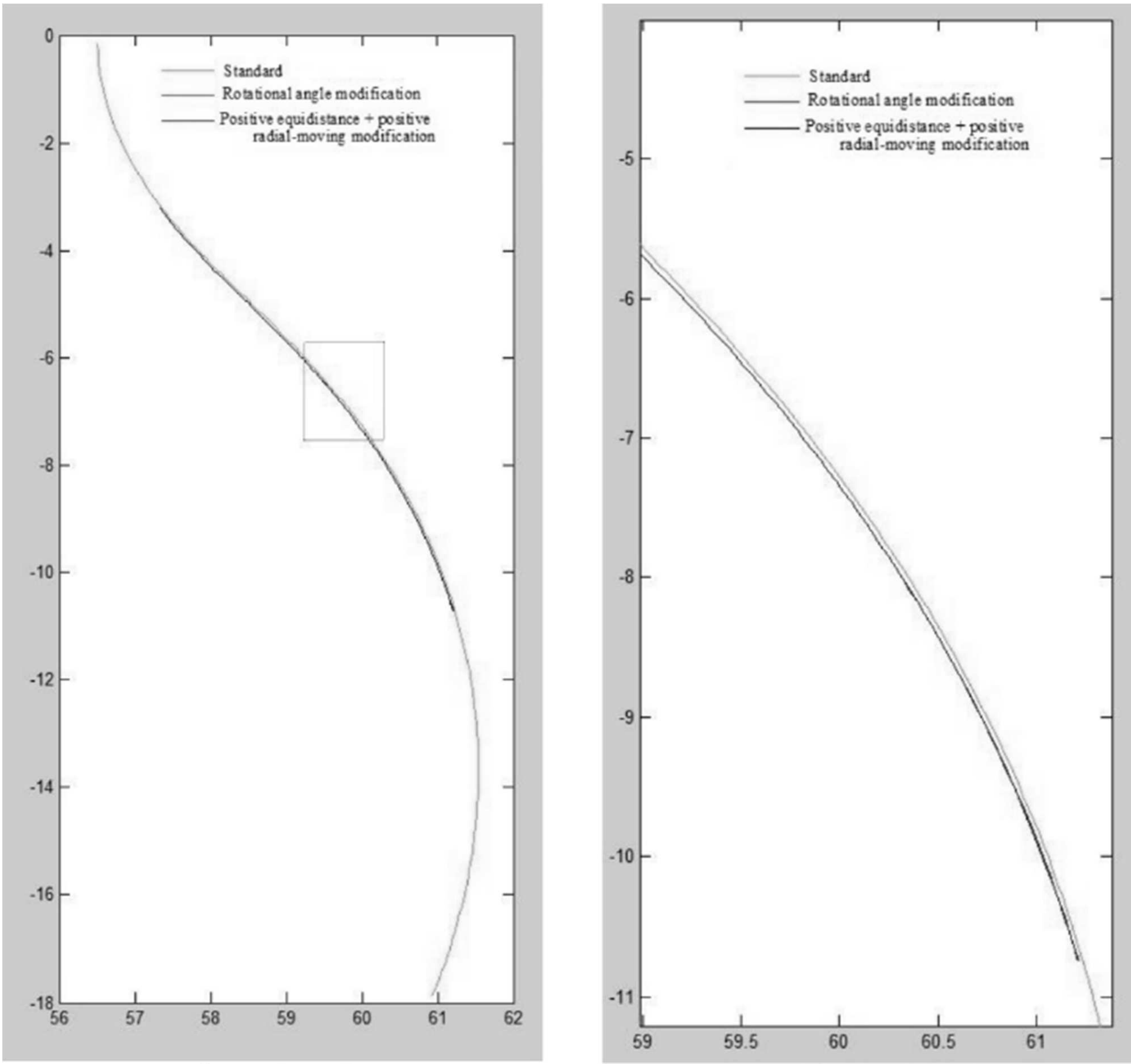
It was supposed that the number of needle teeth $z_p = 12$, the number of cycloid gears $z_c = 11$, the eccentricity $a = 3.5\text{mm}$ mm, the needle tooth distribution circle radius $r_p = 65\text{mm}$, and the radius of needle teeth $r_{rp} = 5\text{mm}$.

Under the premise of ensuring the same side clearance $\Delta c = 0.02\text{mm}$, without considering the backlash, the optimization toolbox was used to solve the objective function. $F_1(\Delta r_{rp}, \Delta r_p)$ The data in Table 1 is the optimization results.

Tab. 1 *The results of backlash and deviation of different modification method*

Modification method		Rotation angle δ (°)	Equidistance Δr_{rp}	Radial-moving Δr_p	Deviation F_1	Backlash $\Delta\varphi(')$
Equidistance	Radial-moving					
"+"	"+"	0.05	0.0766	0.0566	0.0971	5.97
"+"	"–"		0	0.02	1.0954	2.73
"–"	"–"		0	–0.02	1.5123	2.73

Tab. 1 shows that the deviation between the tooth profile with rotation angle modification and the tooth profile with combined modification of "+"equidistant and "+" radial-moving was 0.0971, which was the smallest. As shown in Fig. 4, the curves are comparison diagrams of the tooth profile. The local enlarged view of Fig. 4b shows that, in the working section, the tooth profile with combined modification of "+" equidistant and "+" radial-moving was in good agreement with the rotation angle modified tooth profile, but the calculation using Equation (8) shows that the backlash caused by this kind of combination modification was the largest, which was 5.97′.



(a) 1 / 2 tooth profile (b) partial enlarged drawing

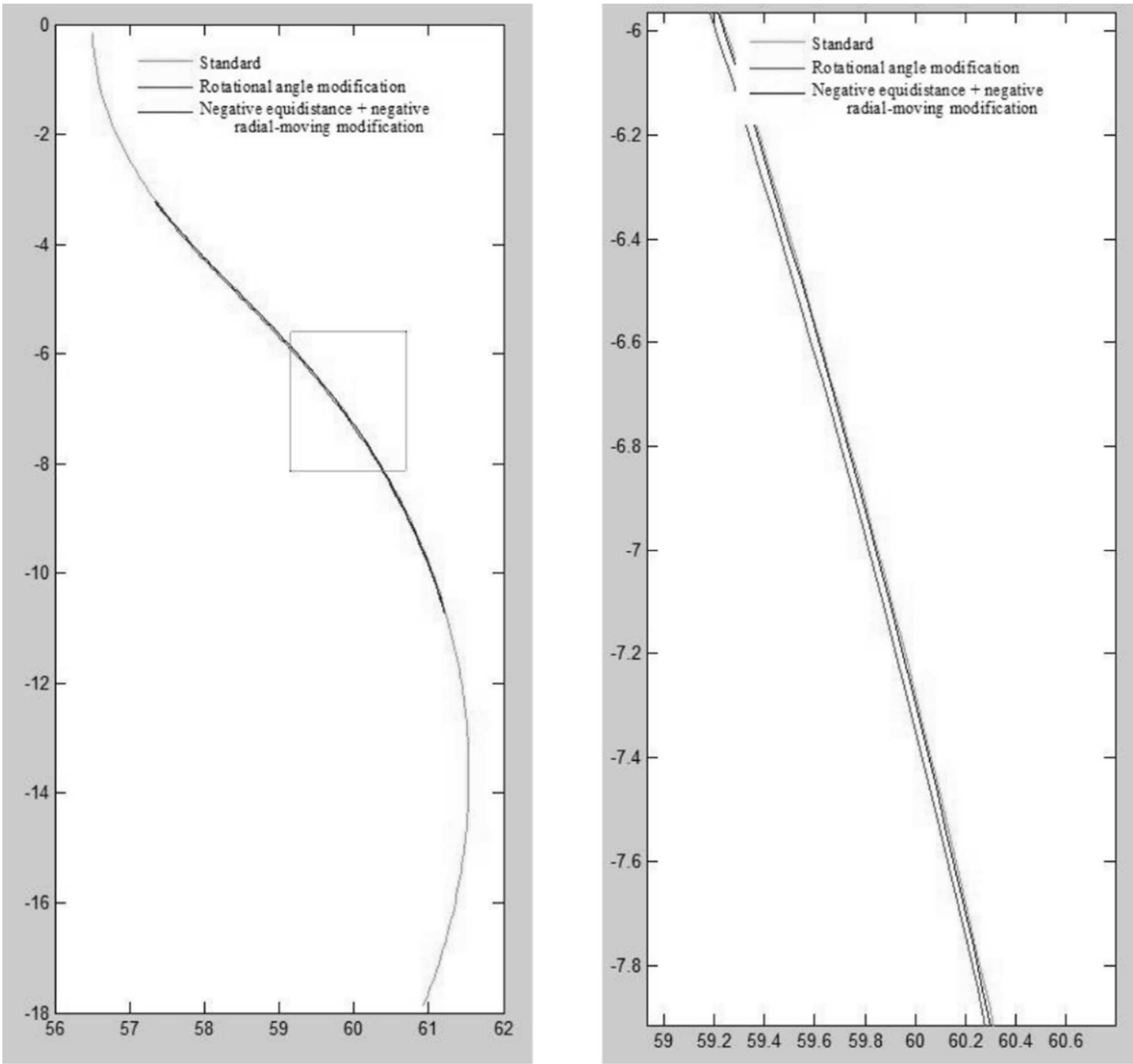
***Fig. 4** Comparison of profile modification with positive equidistance + positive displacement*

Considering the load-carrying capacity and backlash at the same time, the optimal backlash target value was 0.5′, which was solved using the multi-objective nonlinear constrained function of the optimization tool. The optimization results are shown in Tab. 2.

***Tab. 2** The results of backlash and deviation of different modification method*

Modification method		Rotation angle $\delta(°)$	Equidistance Δr_{rp}	Radial-moving Δr_p	Deviation F_1	Backlash $\Delta\varphi(')$
Equidistance	**Radial-moving**					
"+"	"+"	0.05	0.02	0	1.0954	3.5717
"+"	"–"		0.02	0	1.0954	3.5717
"–"	"–"		–0.0456	–0.0656	2.4624	0.7925

Tab. 2 and Fig. 5 show that the deviation between the rotation angle modification and the combination of "-" radial-moving and "-"equidistance modification was large, the walue was 2.4624. However, the resulting backlash was 0.7925′, which was much smaller than that obtained using other combined modification methods.



(a) 1 / 2 tooth profile *(b) partial enlarged drawing*

Fig. 5 *The combined modification of negative equidistance and negative radial-moving*

In the calculation, different side clearance targets were set, including the requirement of increasing the radial clearance $\Delta c\left(\Delta r_{rp}-\Delta r_{p}=\Delta c\right)$. Different combined modifications were calculated using the optimization model. The results were as follows.

- (1) The combination of positive equidistance and positive radial-moving modifications resulted in a tooth profile that was closest to the desired profile with rotation angle modification. However, the resulting backlash was the largest. Hence, this kind of combined modification could be used in a large load cycloid drive with low accuracy requirements.
- (2) The combined modification of positive equidistance and negative radial-moving could not ensure the profile to be close with the conjugate profile and could not meet the requirements of small backlash. Hence, it was generally not used.

- (3) When the combined modification of negative equidistance and negative radial-moving was used, a certain deviation occurred between the combined modification profile and the profile with rotation angle modification, but the backlash was the smallest. Therefore, this kind of combined modification could be adopted by the high-precision cycloid drive.

4 Conclusions

The tooth profile of cycloid transmission, as a multi-tooth mesh transmission, is generally modified during gear grinding to ensure high accuracy and small return difference, and also to meet the lubrication requirements and compensate the manufacturing assembly error. To get proper modified tooth profile, an optimization mathematical model was established through analyzing the influence of cycloid gear tooth modification on cycloid transmission backlash. The tooth modification parameters that ensured the accuracy and strength were obtained through the example of optimization calculation, and the rationality of the optimization method was verified. The analysis showed that the combined modification of positive equidistance and positive radial-moving could approach the angle-modified tooth profile to the greatest extent, but the resulting backlash was also the largest. The combination of negative equidistance and negative radial-moving caused the least backlash. It was recommended to be used in the case of high precision and small backlash.

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References

[1] VALENTE, A., BARALDO, S., CARPANZANO, E., (2017). Smooth Trajectory Generation for Industrial robots Performing High Precision Assembly Processes. *CIRP Annals: Manufacturing Technology*, Vol. 66, No. 1, pp. 17-20.

[2] JANG, D.J., KIM, Y.C., HONG, E.P., KIM, G.S., (2021). Geometry design and dynamic analysis of a modified cycloid reducer with epitrochoid tooth profile. *Mechanism and Machine Theory*, Vol. 164, pp. 104399.

[3] ZHAO, B., DENG, X., LI, T., ET AL. (2015). Theoretical research on the cycloid wheel modification of robot RV Reducer. *Mechanical transmission*, Vol. 12, pp. 1-6.

[4] WANG, H., DENG, X., YANG, J., ET AL. (2014). Forming grinding and test of segmented topological modified gear. *Journal of Aeronautical power*, Vol. 29, No. 12, pp. 3000-3008.

[5] SUN, B., DU, H., WANG, J., ET AL. (2018). Research on Key Technologies of online detection of cycloid wheel of robot RV Reducer. *Journal of sensing technology*, Vol. 31, No. 05, pp. 18-24

[6] CHU, X., XU, H., WU, X., TAO, J., SHAO, G., (2018). The Method of Selective Assembly for the RV Reducer Based on Genetic Algorithm. *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*, Vol. 232, No. 6, pp. 921-929.

[7] FRACHE, L., KOMBA, E.H., PHILIPPON, D., GALIPAUD, J., DE BARROS, M.I., DOUILLARD, T., MASENELLI-VARLOT, K., BOUSCHARAIN, N., MAHEO, Y., SARLIN, R., LE JEUNE, G., (2021). Observation of a modified superficial layer on heavily loaded contacts under grease lubrication. *Tribology International*, Vol. 158, pp. 106921.

[8] BEDNARCZYK, S., (2021). Analysis of the cycloidal reducer output mechanism while taking into account machining deviations. Proceedings of the Institution of Mechanical Engineers, Part C: *Journal of Mechanical Engineering Science*, Vol. 235, No. 23, pp. 7299-7313.

[9] HSIEH, C.F., (2015). Traditional Versus Improved Designs for Cycloidal Speed Reducers with a Small Tooth Difference: the Effect on Dynamics. *Mechanism and Machine Theory*, Vol. 86, pp. 15-35.

[10] XU, L., YANG, Y., (2016) A General Method for Multi-tooth Contact Analysis of Cycloid-pin Gear Transmission. *China Mechanical Engineering*, Vol. 27, No. 10, pp. 1382-1388.

[11] HONG-LIU, Y., JIN-HUA, Y., XIN, H., PING, S., (2013). Study on Teeth Profile Modification of Cycloid Reducer Based on

Non-Hertz Elastic Contact Analysis. *Mechanics Research Communications*, Vol. 48, pp. 87-92.

[12] REN, Z.Y., MAO, S.M., GUO, W.C., GUO, Z., (2017). Tooth Modification and Dynamic Performance of the Cycloidal Drive. *Mechanical Systems and Signal Processing*, Vol. 85, pp. 857-866.

[13] YANG, Y., ZHOU, G., CHANG, L., CHEN, G., (2021). A modelling approach for kinematic equivalent mechanism and rotational transmission error of RV reducer. *Mechanism and Machine Theory,* Vol. 163, No. 1, pp. 104384-104400.

[14] HE, W., SHAN, L., (2015). Research and Analysis on Transmission Error of RV Reducer Used in Robot. *Mechanisms and Machine Science*, Vol. 33, pp. 231-238.

[15] GUAN, T., ZHANG, D., (2005). Research and optimization design of reverse bow tooth profile in cycloid pin wheel planetary drive. *Journal of mechanical engineering,* Vol. 41, No. 1, pp. 151-156.

[16] SU, J., KE, Q., DENG, X., REN, X., (2018). Numerical simulation and experimental analysis of temperature field of gear form grinding. *The International Journal of Advanced Manufacturing Technology,* Vol. 97, pp. 2351-2367.

[17] KOČIŠKO, M., POLLÁK, M., TÖRÖKOVÁ, M., BARON, P., PAULIŠIN, D., KUNDRÍK, J., (2021). Determination of methodology and research of the influence of the trial run of high-precision reducers on the change of their characterizing properties. *Applied Sciences*, Vol. 11, No. 9, pp. 3859-3881.

[18] WANG, R., GAO, F., LIU, R., (2018). Modelling and compensation of cycloid tooth profile of RV Reducer. *Journal of instrumentation*, Vol. 3, pp. 81-88.

[19] WANG, J., LUO, S., SU, D., (2016). Multi-objective Optimal Design of Cycloid Speedreducer Based on Genetic Algorithm. *Mechanism and Machine Theory*, Vol. 102, pp. 135-148.

[20] ZHANG, L., ZHOU, H., WANG, C., ET AL. (2017). Analysis of meshing characteristics and return clearance of precision cycloid drive for robot. *Mechanical transmission*, Vol. 03, pp. 50-53.

[21] BO, W., JIAXU, W., GUANGWU, Z., RONGSONG, Y., HONGJUN, Z., TAO, H., (2016). Mixed Lubrication Analysis of Modified Cycloidal Gear Used in the RV Reducer. *Proceedings of the Institution of Mechanical Engineers, Part J: Journal of Engineering Tribology*, Vol. 230, No. 2, pp. 121-134.

[22] ZHANG, L., ZHENG, T., LI, T., WANG, J., WANG, C., JIANG, Y., LI, C., YUAN, F., YAO, Z. 2023. Precision Forming Process Analysis and Forming Process Simulation of Integrated Structural Gear for New Energy Vehicles. *Manufacturing Technology*, 23, 958-966.,

[23] KOVÁČIKOVÁ, P., DUBEC, A., KURICOVÁ, J. 2021. The Microstructural Study of a Damaged Motorcycle Gear Wheel. *Manufacturing Technology*, 21, 83-90.

[24] CHANG, Z., HU, L. 2021. Damage assessment of the rolling bearing based on the rigid-flexible coupling multi-body vibration model. *Manufacturing Technology*, 21, 340-348.

[25] SUN, X., HAN, L., WANG, J., (2019). Tooth modification and loaded tooth contact analysis of China Bearing Reducer. *ARCHIVE Proceedings of the Institution of Mechanical Engineers Part C Journal of Mechanical Engineering Science* 1989-1996, Vols. 203-210, pp. 095440621985818.