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Research on Linkage Accuracy of "Baby Hanging Basket" Five-axis Precision Machine Tool

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The "baby hanging basket" five-axis precision machine tool (BHBFPMT) studied in this study is mainly used to process open impeller and closed impeller in aerospace field. However, the linkage accuracy (LA) measurement of five-axis machine tool is one of the important detection means to ensure the tolerance requirements of precision parts. The mathematical model of linkage accuracy (LA) of "baby hanging basket" five-axis precision machine tool (BHBFPMT) was established. The accuracy of X, Y, Z, B and C axes was measured, respectively. Combined with the measurement results, a pulse compensation method was proposed to compensate the accuracy. The linkage accuracy of the machine tool was analyzed by the machining and precision measurement method of "round-rhombic-square" trial cutting parts. The results show that: (1) In single-axis measurement, the positioning accuracy of X, Y, Z, B and C axes are 0.00096 mm, 0.00261 mm, 0.00173 mm, 0.0113 mm and 3.62", respectively. (2) When the pulse changes of B axis and C axis are 1048630 and 1048596, respectively, the positioning accuracy meets the accuracy requirements of "baby hanging basket" five-axis precision machine tool. (3) By measuring and analyzing the accuracy of "round-rhomb-square" test pieces, the linkage accuracy of "baby hanging basket" five-axis precision machine tool is less than 20 μm, which meets the factory standards and precision requirements of machine tool.

Keywords: "Baby hanging basket" five-axis precision machine tool (BHBFPMT), Linkage accuracy (LA), Precision, Measurement, Pulse compensation

1 Introduction

Five-axis precision machining center is mainly used to process complex precision parts, while "baby hanging basket" five-axis precision machine tools (BHBFPMT) are mainly used to process open impellers and closed impellers in aerospace field. The stability of BHBFPMT is one of the important factors to ensure the machining accuracy of parts. Many scholars have studied the accuracy calculation and measurement of machine tools.

For example, Li et al. [1] established the motion model of five-axis machine tool linkage accuracy, and measured the geometric error of the machine tool. Based on the measurement conclusion of geometric error, the identification method of geometric error was put forward. Jozwik et al. [2] discussed the test sequence and the evaluation of the motion center position of the rotation axis relative to the linear X and Y axes, and made measurements by using R-test calibration and measurement system. Breitzke et al. [3] put forward a new method to quickly identify geometric errors of three-axis machine tools by on-machine measurement. This method was especially suitable for periodic control of machine tool accuracy to ensure long-term machining accuracy. Wang et al. [4] developed a five-axis machine tool prototype for machining aero-engine shell, and carried out simulation and machining experiments. The results show that the machine tool has good machining accuracy, rigidity and efficiency, and can meet the machining requirements of aero-engine shell. Wang et al. [5] proposed a parametric double spline interpolation method for continuous machining of five-axis machine tools. Compared with the traditional three-dimensional model analysis method of S-shaped specimen, the contour error calculated by this method does not include theoretical error, thus eliminating the influence of theoretical error on the measurement and evaluation of machine tool accuracy. Vahebi et al. [6] proposed a volume error modeling method for five-axis machine tools. This method was helpful to shaft design and structure configuration in the process of error propagation. Li et al. [7] put forward a new method to compensate the volume positioning error of five-axis machine tools. This method can improve the machining accuracy of the machine tool at a very low cost. Xu et al. [8] analyzed the geometric characteristics of S-shaped specimens and processed S-shaped specimens in UG CAM environment, and studied their influence on the accuracy of each axis of five-axis NC machine tools. Xing et al. [9] proposed a machine tool precision condition monitoring scheme using volume error (VEs), vector similarity measure (VSMs) and exponential weighted moving average (EWMA) control chart. Xu et al. [10] put forward a method of machining the end face of cycloidal gear on machine tool by using ball-end milling cutter. The machining accuracy of cycloidal gear was measured by using CMM, and the surface quality of machined parts is improved. Gu et al. [11] proposed to add local offset to the global offset method in five-axis machine tools, and further minimize the feature error of parts by iterative and incremental methods. The verification results show that the residual error of part feature error is less than 10% of feature tolerance. The research on error compensation was typical in references [12-14].

Based on the developed "baby hanging basket" five-axis precision machine tool (BHBFPMT), the linkage accuracy (LA) was studied. The theoretical model of LA error of BHBFPMT was established. Single precision measurement of five axes was carried out by using Renishaw laser interferometer LX80 and the results were analyzed. The measurement results were used and uniaxial pulse compensation was carried out. Finally, the trial cutting and precision detection were carried out through "S" shaped trial cutting pieces and "round-rhombic-square" shaped trial cutting pieces. Therefore, the LA of BHBFPMT was analyzed by measuring the precision of trial cutting parts. In order to ensure the machining accuracy of open impeller and closed impeller, and provide research basis for more scholars to study the LA of five-axis machine tools.

2 Mathematical model of actual linkage accuracy of "baby hanging basket" five-axis precision machine tool.

To study the linkage accuracy of machine tools, it is necessary to analyze the actual motion model of machine tools theoretically. The difference between the actual kinematics model of five-axis CNC machine tool and the ideal kinematics model is that the geometric error of the machine tool is introduced, and the actual kinematics model is the expression of the actual tool position and attitude in the workpiece coordinate system [1] [15]. In the establishment of the actual kinematics model, not only the geometric error of the machine tool [16], but also the thermal error and machining error of the machine tool [17] also have an impact on the actual kinematics model. Because the stiffness of the joint surface between the motorized spindle, the tool shank and the tool changes step by step, the error caused by deflection deformation also exists in the stress process, but these factors are tiny relative to the geometric error of the five motion axes of the machine tool, which is ignored here. The motion axis relationship of the "baby hanging basket" five-axis precision machine tool studied in this paper is shown in Figure 1.

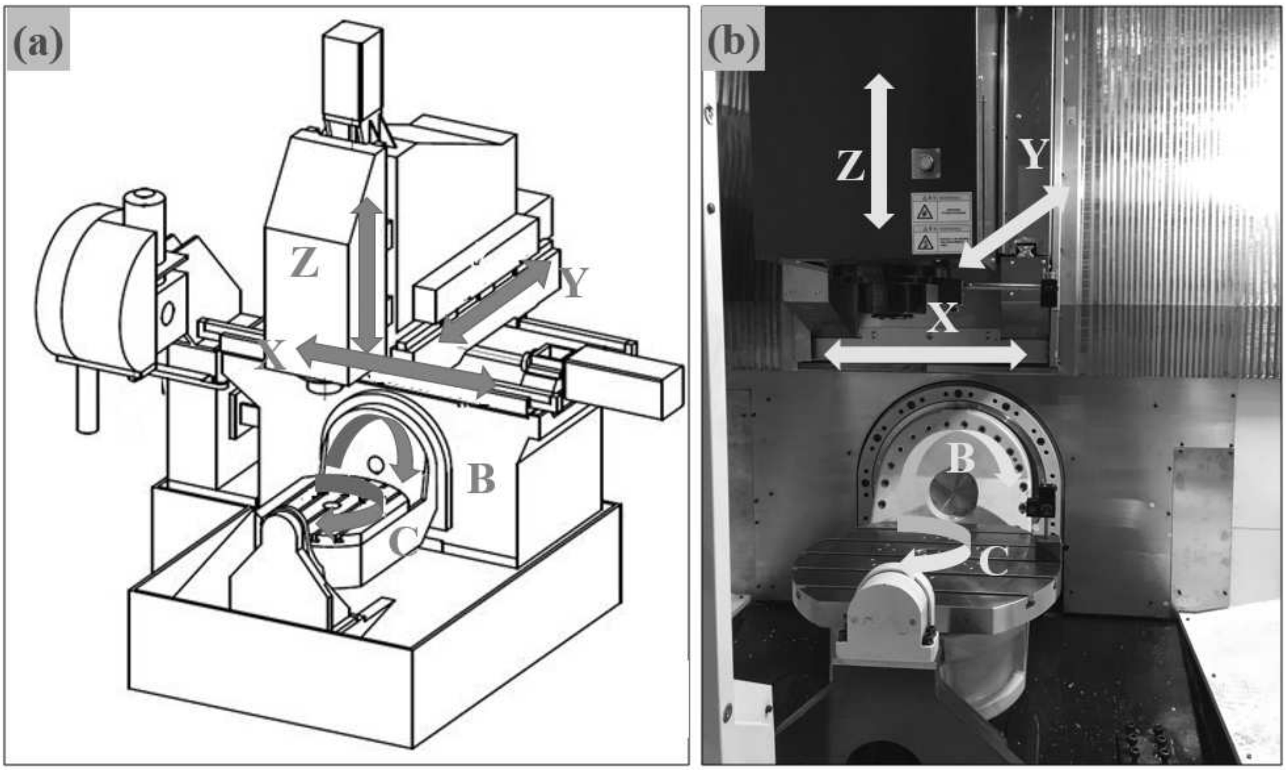


Fig. 1 *Kinematic relationship of "baby hanging basket" five-axis precision machine tool*

As shown in Fig. 1, the X, Y and Z axes belonge to the moving axes, and their strokes are 700 mm, 450 mm and 400 mm, respectively. Axis B and C belong to rotational axes, and their rotational ranges are 110 and 360, respectively. The diameter of the worktable is 650 mm. The maximum speed of the spindle is 12000 rpm. According to Fig. 1 and establish the geometric error elements of the BHBFPMT, as shown in Tab. 1.

Tab. 1 *Geometric error elements of "baby hanging basket" five-axis precision machine tool*

Axis	Positioning error	Linear/position error	Angular error	Verticality error	Parallelism error
X	δ_{xx}	$\delta_{yx}\delta_{zx}$	$\varepsilon_{xx}\varepsilon_{yx}\varepsilon_{zx}$	S_{xy}	
Y	δ_{yy}	$\delta_{zy}\delta_{xy}$	$\varepsilon_{yy}\varepsilon_{zy}\varepsilon_{xy}$	S_{yz}	
Z	δ_{zz}	$\delta_{xz}\delta_{yz}$	$\varepsilon_{zz}\varepsilon_{xz}\varepsilon_{yz}$	S_{zx}	
B		$\delta_{xb}\delta_{yb}\delta_{zb}$	$\varepsilon_{\alpha b}\varepsilon_{\beta b}\varepsilon_{\gamma b}$		$\phi_{yb}\phi_{zb}$
C		$\delta_{xc}\delta_{yc}\delta_{zc}$	$\varepsilon_{\alpha c}\varepsilon_{\beta c}\varepsilon_{\gamma c}$		$\phi_{yc}\phi_{xc}$

According to Fig. 1 and Tab. 2, the coordinate transformation relationship between each motion axis unit can be obtained. The actual motion transformation matrix of the B-axis local motion coordinate system O_B relative to the original machine tool coordinate system O_R was shown in Eq. (1).

$$ {}^{R}\tilde{R}_{B} = \begin{bmatrix} 1 & -\varepsilon_{yb} & \varepsilon_{\beta b} & \delta_{xb} \\ \varepsilon_{yb} & 1 & \varepsilon_{\alpha b} & \delta_{yb} \\ -\varepsilon_{\beta b} & -\varepsilon_{\alpha b} & 1 & \delta_{zb} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos b & -\sin b & 0 \\ 0 & \sin b & \cos b & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1} $$

The "~" superscript was used to represent the actual transformation matrix, and the actual motion transformation matrix of C-axis local motion coordinate system O_C relative to A-axis local motion coordinate system O_B was Eq. (2).

$$ {}^{R}\tilde{R}_{C} = \begin{bmatrix} 1 & 0 & S_{\beta cc} & 0 \\ 0 & 1 & 0 & \delta_{ycc} \\ -S_{\beta cc} & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -\varepsilon_{yc} & \varepsilon_{\beta c} & \delta_{xc} \\ \varepsilon_{yc} & 1 & \varepsilon_{\alpha c} & \delta_{yc} \\ -\varepsilon_{\beta c} & -\varepsilon_{\alpha c} & 1 & \delta_{zc} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos c & -\sin c & 0 & 0 \\ \sin c & \cos c & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2} $$

The actual motion transformation matrix of the workpiece coordinate O_W relative to the C-axis local motion coordinate system O_C was identical to the ideal one as Eq. (3).

$$ {}^{C}L_{W} = \begin{bmatrix} 1 & 0 & 0 & w_x \\ 0 & 1 & 0 & w_y \\ 0 & 0 & 1 & w_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3} $$

Similarly, the actual motion transformation matrix of the tool coordinate system O_T relative to the original machine tool coordinate system O_R passes through four motion coordinate systems: spindle coordinate system O_S, Z-axis coordinate system O_Z, Y-axis coordinate system O_Y and X-axis coordinate system O_X. Then the actual motion transformation matrix of the tool coordinate system O_T relative to the original machine tool coordinate system O_R was superimposed with the error matrix, as shown in Eq. (4).

$$ {}^{R}\tilde{L}_{T} = {}^{R}\tilde{L}_{X} \cdot {}^{X}\tilde{L}_{Y} \cdot {}^{Y}\tilde{L}_{Z} \cdot {}^{Z}\tilde{L}_{S} \cdot {}^{S}\tilde{L}_{T} \tag{4} $$

$$ {}^{Y}\tilde{L}_{Z} = \begin{bmatrix} 1 & 0 & S_{\beta zy} & 0 \\ 0 & 1 & -S_{\alpha zy} & 0 \\ -S_{\beta zy} & S_{\alpha zy} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -\varepsilon_{\gamma z} & \varepsilon_{\beta z} & \delta_{xz} \\ \varepsilon_{\gamma z} & 1 & \varepsilon_{\alpha z} & \delta_{yz} \\ -\varepsilon_{\beta z} & -\varepsilon_{\alpha z} & 1 & \delta_{zz} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{5} $$

$$ {}^{X}\tilde{L}_{Y}=\begin{bmatrix}1&0&S_{\beta yx}&0\\0&1&-S_{\alpha yx}&0\\-S_{\beta yx}&S_{\alpha yx}&1&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}1&-\varepsilon_{\gamma y}&\varepsilon_{\beta y}&\delta_{xy}\\\varepsilon_{\gamma z}&1&\varepsilon_{\alpha y}&\delta_{yy}\\-\varepsilon_{\beta y}&-\varepsilon_{\alpha y}&1&\delta_{zz}\\0&0&0&1\end{bmatrix}\begin{bmatrix}0&0&0&0\\0&0&0&y\\0&0&1&0\\0&0&0&1\end{bmatrix} \tag{6} $$

$$ {}^{R}\tilde{L}_{X}=\begin{bmatrix}1&-\varepsilon_{\gamma x}&\varepsilon_{\beta x}&\delta_{xx}\\\varepsilon_{\gamma x}&1&\varepsilon_{\alpha x}&\delta_{yx}\\-\varepsilon_{\beta x}&-\varepsilon_{\alpha x}&1&\delta_{zx}\\0&0&0&1\end{bmatrix}\begin{bmatrix}0&0&0&x\\0&0&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix} \tag{7} $$

Where, the motion quantities of each axis of the BHBFPMT are x, y, z, b and c, respectively, and the initial offset of the workpiece coordinate system O_W in three directions relative to O_C is w_x, wy and wz. S is perpendicularity. The following two subscripts represent the perpendicularity between the two axes, and the previous subscript represents the deviation about a certain axis. Based on the derivation of Eq.s (1)- (7), according to the system error kinematic chain of C axis-B axis-machine tool coordinate system, and combined with the kinematic transformation matrix, the coordinate transformation model of tool position and attitude in workpiece coordinate system in actual state was established, as shown in Eq.s (8) and (9).

$$ {}^{R}\tilde{L}_{X}\cdot{}^{X}\tilde{L}_{Y}\cdot{}^{Y}\tilde{L}_{Z}\cdot{}^{Z}\tilde{L}_{S}\cdot{}^{S}\tilde{L}_{T}\cdot[0\quad 0\quad 0\quad 1]^{T}={}^{R}\tilde{R}_{B}\cdot{}^{B}\tilde{R}_{C}\cdot{}^{C}\tilde{L}_{W}\cdot\tilde{p}_{w} \tag{8} $$

$$ {}^{R}\tilde{L}_{X}\cdot{}^{X}\tilde{L}_{Y}\cdot{}^{Y}\tilde{L}_{Z}\cdot{}^{Z}\tilde{L}_{S}\cdot{}^{S}\tilde{L}_{T}\cdot[0\quad 0\quad 0\quad 1]^{T}={}^{R}\tilde{R}_{A}\cdot{}^{B}\tilde{R}_{C}\cdot{}^{C}\tilde{L}_{W}\cdot\tilde{v}_{w} \tag{9} $$

Where, $\tilde{p}_w$ and $\tilde{v}_w$ are the actual position and posture of the tool in the workpiece coordinate system. The positions and postures $\tilde{p}_w$ and $\tilde{v}_w$ in Eq.s (8) and (9) were extracted and arranged to obtain Eq.s (10) and (11).

$$ \tilde{p}_{w}=({}^{R}\tilde{R}_{A}\cdot{}^{A}\tilde{R}_{C}\cdot{}^{C}\tilde{L}_{W})^{-1}\cdot{}^{R}\tilde{L}_{X}\cdot{}^{X}\tilde{L}_{Y}\cdot{}^{Y}\tilde{L}_{Z}\cdot{}^{Z}\tilde{L}_{S}\cdot{}^{S}\tilde{L}_{T}\cdot[0\quad 0\quad 0\quad 1]^{T} \tag{10} $$

$$ \tilde{v}_{w}=({}^{R}\tilde{R}_{A}\cdot{}^{B}\tilde{R}_{C}\cdot{}^{C}\tilde{L}_{W})^{-1}\cdot{}^{R}\tilde{L}_{X}\cdot{}^{X}\tilde{L}_{Y}\cdot{}^{Y}\tilde{L}_{Z}\cdot{}^{Z}\tilde{L}_{S}\cdot{}^{S}\tilde{L}_{T}\cdot[0\quad 0\quad 0\quad 1]^{T} \tag{11} $$

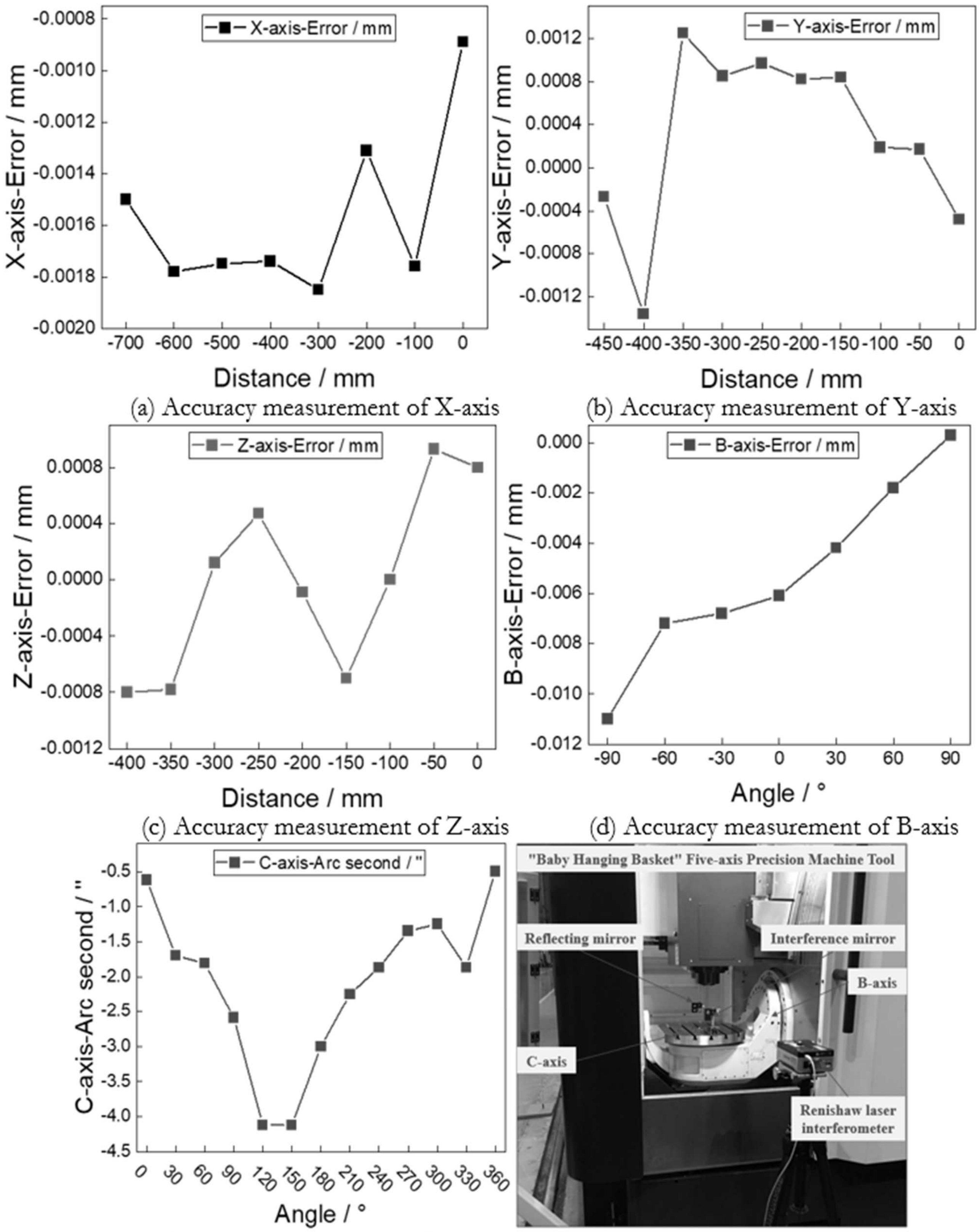
Eq.s (10) and (11) were the actual LA conversion model of the BHBFPMT.

3 Single axis precision measurement and compensation

Based on the derivation of the actual LA theory of the BHBFPMT, the single-axis precision measurement of the BHBFPMT was carried out by using Renishaw laser interferometer XL80. Since the repeated positioning accuracy meets the requirements of the required machine tool (≤ 4 μ m), this study only listed the average positioning accuracy results after 5 measurements, as shown in Fig. 2.

In Fig. 2, (f) was a schematic diagram of accuracy measurement. As shown in Fig. 2(a), the upper and lower limits of positioning accuracy for the X axis were -0.00185 mm and -0.00089 mm, respectively, so the positioning accuracy was 0.00096 mm. As shown in Fig. 2(b), the upper and lower limits of the positioning accuracy for the Y axis wre 0.00125 mm and -0.00136 mm, respectively, so the positioning accuracy was 0.00261 mm. As shown in Fig. 2(c), the upper and lower limits of the Z-axis positioning accuracy were 0.00093 mm and -0.0008 mm, respectively, so the positioning accuracy was 0.00173 mm. As shown in Fig. 2(d), the upper and lower limits of the positioning accuracy for the B axis were 0.0003 mm and -0.011 mm, respectively, so the positioning accuracy was 0.0113 mm. As shown in Fig. 2(e), the upper and lower limits of positioning accuracy for the C axis were -0.5 "and-4.12", respectively, so the positioning accuracy was 3.62 ".

For the above results, the positioning accuracy of linear axis X, Y and Z were within 3 μm, which meets the positioning accuracy requirements of the BHBFPMT. However, the positioning accuracy of B-axis and C-axis were high, so single-axis compensation was needed. There were reverse clearance, pitch, pulse compensation and mechanical compensation in machine tool compensation. However, the ultimate goal of each compensation was to meet the accuracy requirements of machine tools. In this study, we will choose pulse compensation, which was a rare way. Because pulse compensation needs to calculate a lot of data, it is much more difficult than reverse clearance, pitch and mechanical compensation, but it is more accurate. The BHBFPMT studied in this study had a single axis and a single pitch corresponding to 1048576 pulses. Because this study focuses on the linkage accuracy of the BHBFPMT, the calculation process is not listed in detail. A large number of calculations and tests are carried out on the number of pulses on B axis and C axis. When it is 1048630 and 1048596 after correction, the positioning accuracy meets the accuracy requirements of the BHBFPMT.



(a) Accuracy measurement of X-axis (b) Accuracy measurement of Y-axis

(c) Accuracy measurement of Z-axis (d) Accuracy measurement of B-axis

(e) Accuracy measurement of C-axis (f) Schematic diagram of measurement

Fig. 2 *Average measurement results of single-axis precision of "baby hanging basket" five-axis precision machine tool*

4 Trial cutting and linkage accuracy measurement

When the single-axis repeated positioning accuracy and positioning accuracy meet the accuracy requirements of the BHBFPMT, the further five-axis linkage test will be carried out, as shown in Fig. 3.

In the LA measurement of the BHBFPMT, the five-axis LA was dynamically measured without trial cutting (Fig. 3(a)). When the visual inspection meets the requirements, the machining of "S" shaped trial cutting parts and "round-rhombic-square" shaped trial cutting parts were carried out, and finally the LA of the machine tool was reflected by measuring the accuracy of the trial cutting parts. Because the "S" shaped trial cutting pieces (Fig. 3(b)) and the "round-rhombic-square" shaped trial cutting pieces (Fig. 3(c)) were all international standardized machine tool LA inspection trial cutting pieces, and most machine tool manufacturers and machine tool precision research scholars all adopt the "S" shaped trial cutting pieces. This study mainly analyzed the "round-rhombic-square" shape trial cutting pieces, as shown in Fig. 4. The analysis results using the CMM were shown in Fig. 5.

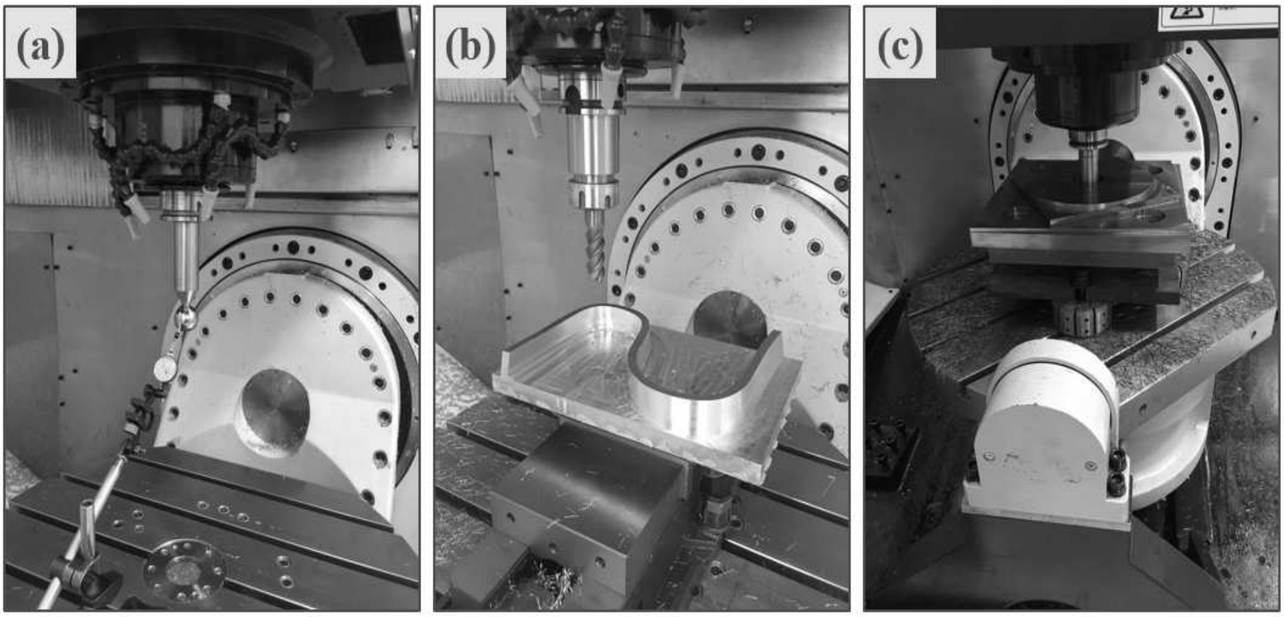


Fig. 3 *Linkage test of "baby hanging basket" five-axis precision machine tool*

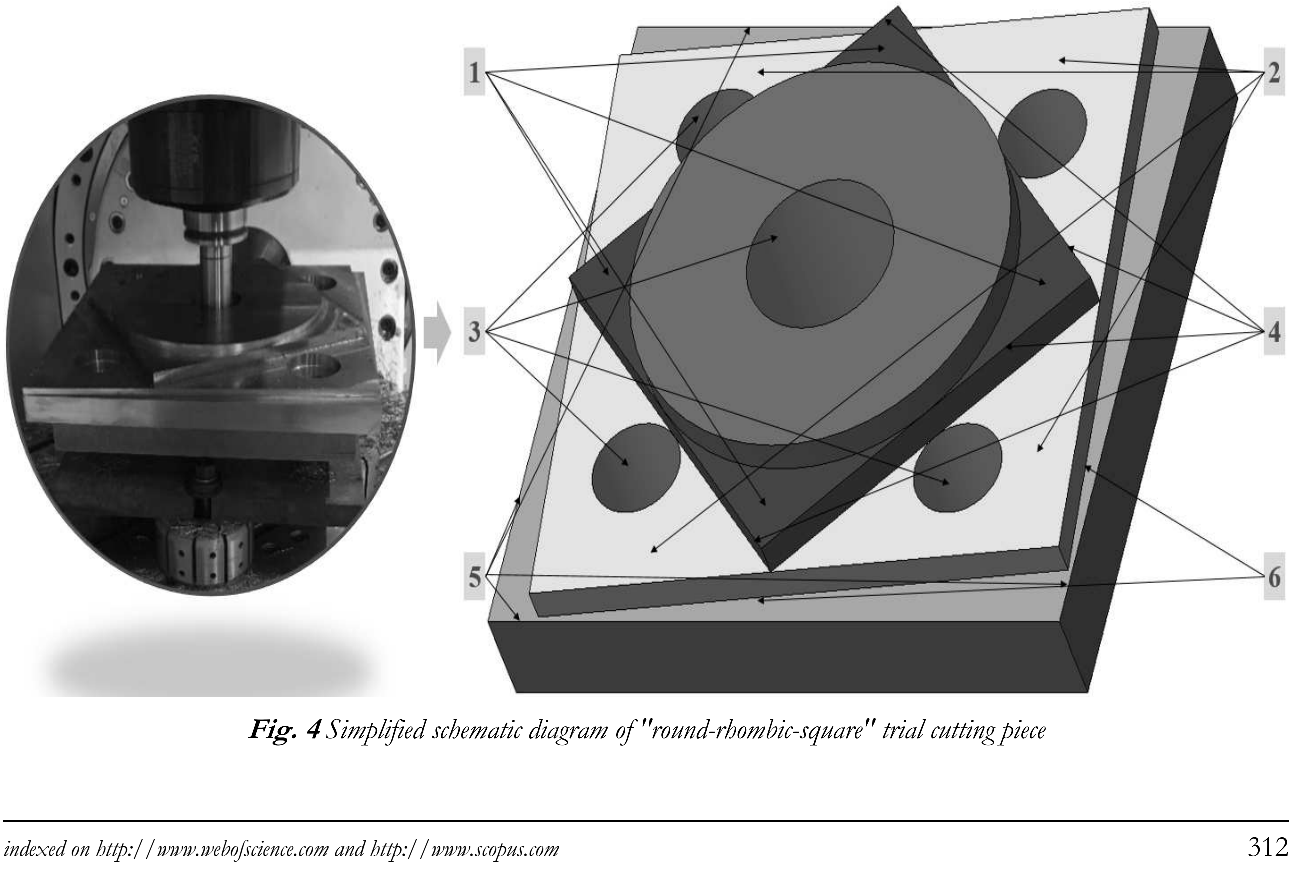


Fig. 4 *Simplified schematic diagram of "round-rhombic-square" trial cutting piece*

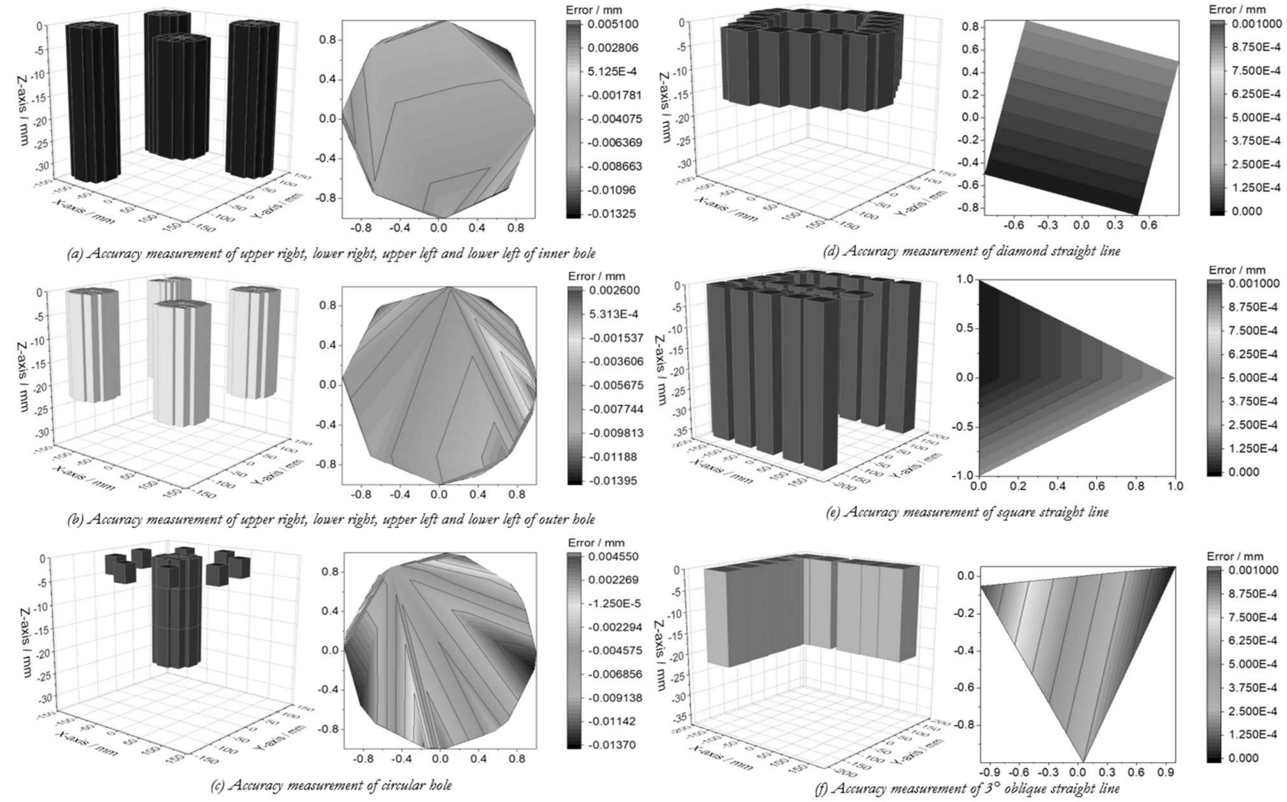


Fig. 5 *Accuracy measurement results of "round-rhombic-square" trial cutting pieces*

According to the analysis of Figs. 4 and 5, the machining positions of numbers 1, 2, 3, 4, 5 and 6 in Fig. 4 correspond to the measurements of Fig. 5(a), (b), (c), (d), (e) and (f), respectively. In Fig. 5(a), the maximum and minimum measurement accuracy of machining position (Fig. 4(1)) were 0.0054 mm and -0.01325 mm, respectively, and the comprehensive accuracy was 0.01865 mm. In Fig. 5(b), the maximum and minimum measurement accuracy of machining position (Fig. 4(2)) were 0.00275 mm and -0.01398 mm, respectively, and the comprehensive accuracy was 0.01673 mm. In Fig. 5(c), the maximum and minimum measurement accuracy of machining position (Fig. 4(3)) were 0.00564 mm and -0.01381 mm, respectively, and the comprehensive accuracy was 0.01945 mm. Meanwhile, in Figs. 5(d), (e) and (f), the combined accuracy of Figs. 4(4), (5) and (6) corresponding to the machining positions were about 1 μm.

Through the above precision measurement and analysis, the LA measurement of the BHBFPMT was less than 20 μm, and meet the factory standards and accuracy requirements of machine tools.

5 Conclusion

In this study, the mathematical model of the BHBFPMT was deduced, and the single-axis measurement of five-axis machine tool was carried out by using Renishaw laser interferometer XL80. According to the measurement results, pulse compensation was carried out. The LA of the BHBFPMT was obtained through the machining of "circle-rhomb-square" trial cutting piece and three-coordinate measurement, and the following conclusions were formed:

- In single-axis measurement, the positioning accuracy of X, Y, Z, B and C axes were 0.00096 mm, 0.00261 mm, 0.00173 mm, 0.0113 mm and 3.62 ", respectively. By calculating the equal pulse number of single axis pitch, when the pulse number of B axis and C axis were 1048630 and 1048596, respectively, the positioning accuracy meet the accuracy requirements of the BHBFPMT.
- The "round-rhombic-square" shape trial-cut piece was selected in the process of trial-cut piece processing. By measuring and analyzing the precision of "round-rhombic-square" type trial cutting parts, the LA measurement of the BHBFPMT was less than 20 μm, and meet the factory standards and accuracy requirements of machine tool.
- In this study, there are two points that few authors have studied: one is the machining and measuring method of "round-rhomb-square" trial cutting parts. Second, pulse

compensation is selected in uniaxial compensation. Of course, regardless of the number of machine tool motion axes, it can also be corrected by pulse compensation, reverse clearance and pitch compensation. Meanwhlie, the three correction methods can also be combined to achieve the accuracy requirements of the machine tool. Therefore, the author hopes that this study can provide a powerful reference for machine tool precision measurement and compensation.

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