DOI: 10.21062/mft.2026.043 © 2026 Manufacturing Technology. All rights reserved. http://www.journalmt.com

Model Solution for Vibrations of Mechanical Systems – Part I

Frantisek Klimenda (0000-0001-7937-3755)[1], Blanka Skocilasova (0000-0001-8242-3231)[1], Josef Soukup (0000-0001-8325-3529), Tereza Jarosova (0000-0002-3651-5079)[2]
[1]Faculty of Mechanical Engineering, J. E. Purkyne University in Usti nad Labem. Pasteurova 3334/7, 400 01 Usti nad Labem. Czech Republic. E-mail: frantisek.klimenda@ujep.cz
[2]High School of Shipping and Technical Crafts, Decin VI, p. o., Delnicka 825/15, Decin VI-Letna, 405 02 Decin, 405 02 Decin VI, Czech Republic

The article deals with the solution of vertical vibrations of mechanical systems (ma-chines). The introduction briefly characterizes the boundary conditions of the pro-posed solution, while the analytical solution itself is based on Lagrange's equation of the second kind. Its solution for this case is briefly outlined. A simplified spatial model of a material system is proposed, which allows changing the distribution of masses and various kinematic excitations of the resulting system. Relationships for the vertical displacement of individual points of the proposed mechanical system are derived for various variants of kinematic excitation. In the next part of the article, an analytical calculation of the vertical displacements of selected points is performed for an un-damped mechanical system with symmetric and asymmetric kinematic excitation. The calculation is performed with a defined mechanical system consisting of a rigid plate with two counterweights that allow the distribution of masses to be changed. The system is mounted on four forced springs (cylindrical) with defined stiffness. Selected results for different arrangements of the mechanical system under different kinematic excitations are presented in the article.

Keywords: Vibration, Mechanical system, Kinematic excitation, Rotation angles

1 Introduction

Vibrations are a common phenomenon in the operation of machines and technical equipment in general. High-intensity vibrations can significantly reduce their service life and operability. Vibrations can also cause accidents or, in more severe cases, complete destruction. These oscillations are also a source of noise in the vicinity of these machines. Vibrations are particularly important in means of transport, where people are part of the system and the vibrations are transmitted to them. These vibrations affect not only the comfort of travel, but also the safety of the vehicle and can even lead to accidents, where vertical vibrations in particular can cause the wheels to lose con-tact with the track (both in road and rail vehicles) [1].

The development of new machines is usually associated with increased performance and, often, larger dimensions. The requirement to save material and reduce the weight of new machines during construction leads to an increase in their dynamic flexibility. This results in a greater number of natural vibration modes and frequencies, as well as an increase in the occurrence of resonance states [1].

The issue of machine vibration is given considerable attention in technical literature. The behavior of machines, or rather their vibration caused by either internal or external influences, is a limiting factor in their design. It can be said that despite the great attention paid to balancing them or reducing forces, whether external or internal, not all problems associated with this have been satisfactorily resolved.

Vibrating mechanical systems can be classified according to a number of criteria. According to [2, 3], from the point of view of mathematical modeling, we can classify them according to:

- degrees of freedom
- type of differential equations
- origin of oscillations
- nature of motion.

Most publications deal with the issue of vertical vibration in wheeled vehicles, where despite considerable effort, a number of problems remain unresolved [1]. Although machine vibration is generally covered in fewer publications, solving this problem remains a topical issue. It is important to note that, especially in the case of heavy machines (turbines, presses, hammers, etc.), it is important to build a sufficiently rigid foundation that must be flexibly mounted. In any dynamic calculation, the first step is to determine the natural frequencies of the system, e.g., [2, 4-7]. It is important that this natural frequency is sufficiently distant in frequency from

the frequencies caused by the forces generated during the machine's operating mode. Vibration is often caused by rotating masses, which we try to balance as best as possible. In this case, it is important that the natural frequency of the machine is sufficiently distant from the so-called critical speed. In addition to the usual calculation method (using circular frequency), this speed can also be determined using the static deformation of springs caused by the weight of the base and the machine [8].

As stated in [2, 9, 10], the most important step in assessing the dynamic properties of machines is the dynamic analysis of their individual components (torsional stress, uneven running, force transmission to the base, stress on load-bearing structures, etc.). The dynamic properties of a machine are based on the mutual movements of its individual parts. These include, in particular, the influence of inertial forces, the stiffness and damping properties of individual machine parts, and the type of kinematic excitation (periodic, aperiodic, transient) [11].

The basis for solving vibrations in machines or mechanical systems is the analysis of their dynamic properties, on the basis of which mathematical models are created [2, 9, 12]. When creating mathematical models, it is necessary to ensure that they are consistent with physical models and that their mathematical solution is as simple as possible.

In terms of the resistance of building structures to dynamic loads from vibrating mechanical systems, these structures can be divided into x classes according to their resistance to vibrations (there are three classes in the Czech Republic). According to this classification, the resistance of buildings is determined by the vibration velocity (in $mm \cdot s^{-2}$) and velocity (in $mm \cdot s^{-1}$) in a given frequency range and the type of loading (occasional impact or periodic vibration causes impact vibration).

Machines are also classified in terms of permissible vibration with regard to their use. According to these criteria, machines can be divided into four sensitivity classes, for example. The division into individual classes is determined according to the limit value of acceleration (in $mm \cdot s^{-2}$) for vibration frequencies (vibration) up to 10 Hz, and velocity (in $mm \cdot s^{-1}$) for frequencies above 10 Hz.

Tab. 1 *Classification of machinery and equipment according to sensitivity class*

Sensitivity class	Characteristic equipment and machines in a given class
I	Highly accurate machines and automatic machines, apparatus for adjusting optical and precision measuring instruments, microscopes and measuring microscopes, interferometers, optometers and other precision optical instruments, measuring instruments for measurements in the order of several μm, etc.
II	Grinding machines for ball bearings, gear cutting machines, precision milling machines and lathes with a tolerance of 0.0× mm, automatic grinding machines for blades and other precision machines.
III	Metalworking machines of normal precision (lathes, milling machines, drills, horizontal machines, grinders, etc.) of normal precision, spinning and weaving machines, printing machines, etc.
IV	Fans, centrifuges, electric motors, punches and presses for metalworking and light industry, drums for various purposes, sewing machines, sorters, vibrating tables, crushers, shakers, etc.

Currently, FEM and simulation methods are mainly used to analyze the vibration of mechanical systems (machines) [13-20]. The use of these computational methods does not diminish the importance of analytical methods, which allow for more accurate identification of vibration parameters. As already mentioned, for analytical solutions, it is important to correctly design a computational model (mathematical and physical) of the real mechanical system (machine) to be solved.

This article deals with the analytical solution of vibrations in a mechanical system (machine).

2 Theoretical solution

When designing machines, it is important to pay attention not only to the dynamics of the machine itself, but also to its dynamic effect on the foundations on which the machine is mounted. When designing a machine or assessing its impact on its surroundings, we start from the movement of a rigid body. In the simplest case, we con-sider a system with three degrees of freedom, but more often with several degrees of freedom, mounted on linear springs (or even with shock absorbers). We generally con-sider the body (machine) as a body with asymmetrically

distributed mass and asymmetrical excitation (usually kinematic). If we consider damping forces in the calculation, we assume them to be proportional to the speed of spring deformation. The calculation of the vibrations of this system leads to differential equations of motion, assuming that the distribution of masses, damping and spring elements, and the action of external forces and moments can be completely general.

Solving machine vibrations usually leads to very complex relationships, which are sometimes unsolvable. Therefore, a methodology based on simple and generally easily solvable analytical models has been proposed, which allows for quick orientation in the given issue and transition to more complex solutions. This method can be applied to both stationary and mobile devices.

The article deals with the analytical calculation of a mechanical system based on a simple model (Figure 1).

When solving the problem, we must respect the effects of asymmetry in the mechanical system being solved with respect to the axes of geometric symmetry, which intersect at the geometric center. At this center, we consider the center of gravity of the system, which, however, deviates from this position due to asymmetry. The analytical solution is based on Lagrange equations of the second kind.

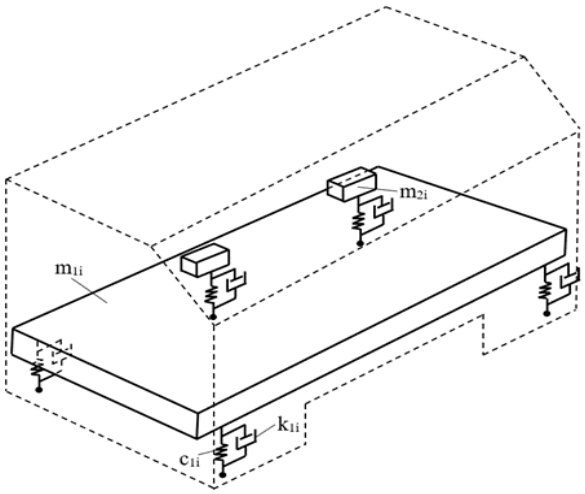


Fig. 1 *Simplified model of a three-dimensional material system*

Where:
m_i…Individual substitute materials;
c_i…Springs;
k_i…Shock absorbers.

$$\frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{q}_j}\right) - \frac{\partial E_k}{\partial q_j} + \frac{\partial E_p}{\partial q_j} + \frac{\partial D}{\partial \dot{q}_j} = Q_j \qquad \text{for } j = 1, 2, \ldots, p \tag{1}$$

Where:
p…Number of degrees of freedom;
q_j…Generalized coordinate;
E_k…Kinetic energy;
E_p…Potential energy;
D…Rayleigh dissipated energy;
Q_j…Excitation forces.

Assuming that the axes of the system under consideration are identical to the main central axes of inertia of the body, the deviation moments $D_{xy} = 0$, $D_{yz} = 0$, $D_{xz} = 0$, the relationship for E_k can be written in simplified form:

$$E_k = \frac{1}{2}\left[m\left(\dot{u}_0^2 + \dot{v}_0^2 + \dot{w}_0^2\right) + J_x\dot{\varphi}_x^2 + J_y\dot{\varphi}_y^2 + J_z\dot{\varphi}_z^2\right] \tag{2}$$

Where:
m…Weight;
$\dot{u}, \dot{v}, \dot{w}$…Velocity in the direction of the axes x, y, z;
J_x, J_y, J_z…Moments of inertia of a body about individual axes;
$\dot{\varphi}_x, \dot{\varphi}_x, \dot{\varphi}_x$ …Angles of rotation of individual axes.

The potential energy E_p in equation (1) can be determined from the relationship:

$$E_p = \sum_{i=1}^{n} E_{pi} = \sum_{i=1}^{n} \left(\frac{1}{2}k_i s_{oi}^2 + \frac{1}{2}\varkappa_i s_{pi}^2\right) \qquad \text{for } n \geq 3 \tag{3}$$

Where:
s_{oi}…Spring displacement in the direction of the longitudinal axis;
k_i…Longitudinal stiffness of the spring;
s_{pi}…Displacement perpendicular to the longitudinal axis of the spring;
$\varkappa_i$…Transverse (shear) stiffness of the spring.

After adjustment, Rayleigh's dissipative function can be determined from the relationship:

$$D = \frac{1}{2}\sum_{i=1}^{n} b_i\left(\dot{u}_i\cos a_i + \dot{v}_i\cos\beta_i + \dot{w}_i\cos\gamma_i\right)^2 \tag{4}$$

Where:

b_i…Linear viscous damping coefficients,

a, β, γ…Angles formed by the damping force carrier with the axes x, y, z.

When investigating the influence of asymmetry in a mechanical system on its vertical vibration, it is appropriate to replace the general simplified model of a spatial mechanical system (Figure 1) with a model that allows for systematic analysis of various cases of mass and geometric asymmetry, including excitation asymmetry. This re-fined model of the actual mechanism (Figure 2) consists of a rigid plate representing the machine frame, including other mechanisms (sprung mass), which is mounted on unspring mass (e.g., foundation) on springs with dampers.

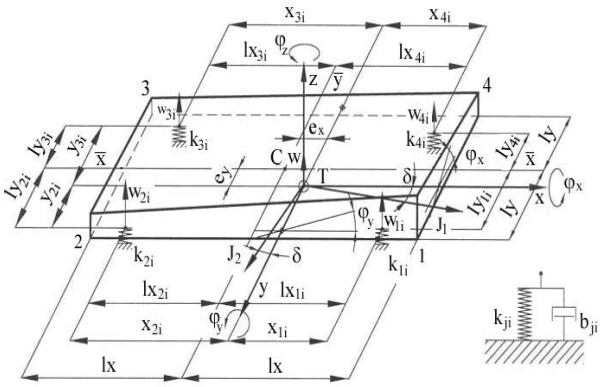


Fig. 2 *Refined spatial model of the material system*

Where:

C…Geometric center;

T…Center of gravity;

e_x, e_y…Eccentricity of the center of gravity;

x, y, z…Coordinate system passing through the center of gravity;

$\bar{x}$, $\bar{y}$, $\bar{z}$ …Axes passing through the geometric center;

φ_x, φ_y…Angles of rotation of axes;

k_i…Spring stiffness;

b_i…Damper stiffness.

In the solution according to Figure 2, two coordinate systems are considered. The first system is given by the geometry of the body, and the origin of the system lies at the geometric center *C*. The second system is given by the distribution of the body's mass and has its origin at the center of gravity *T* of the material system. Its x and y axes are parallel to the axes of the geometric body, i.e. $x \,||\, \bar{x}$ and $y \,||\, \bar{y}$. The same applies to the z axis. The coordinate systems mentioned are left-handed and are divided into four quadrants $j = 1, 2, 3, 4$. Any i-th point, the location of the plate, at which the conservative force $k_i w_i$ and the dissipative force $b_i \dot{w}_i$ act, is determined in the geometric system by coordinates $\bar{x}_i = l_{xji}$ a $\bar{y}_i = l_{yji}$, i.e., $A = (l_{xji}, l_{yji})$.

In the selected coordinate system x, y, z, the position of any point A_i is given by the coordinates:

$$x_i = \pm l_{xji} \pm e_x \qquad y_i = \pm l_{yji} \pm e_y \tag{5}$$

Where:

e_x, e_y…Coordinates of the center of gravity in the $\bar{x}$, $\bar{y}$ system determining the eccentricity of the center of gravity position.

The position of the plate on coil springs and viscous dampers (Figure 2) in individual quadrants is given in the x, y coordinate system with the origin at the center of gravity *T* by the following relationships:

I. quadrant $j = 1$

$$x_{1i} = l_{x1i} - e_x = (a_{1i} - \eta_x) l_x \qquad y_{1i} = l_{y1i} - e_y = (\beta_{1i} - \eta_y) l_y \tag{6a}$$

II. quadrant $j = 2$

$$x_{2i} = -(l_{x2i} - e_x) = -(a_{2i} + \eta_x) l_x \qquad y_{2i} = l_{y2i} - e_y = (\beta_{2i} - \eta_y) l_y \tag{6b}$$

III. quadrant $j = 3$

$$x_{3i} = -(l_{x3i} + e_x) = -(a_{3i} + \eta_x) l_x \qquad y_{3i} = -(l_{y3i} + e_y) = -(\beta_{3i} + \eta_y) l_y \tag{6c}$$

IV. quadrant $j = 4$

$$x_{4i} = l_{x4i} - e_x = (a_{4i} - \eta_x) l_x \qquad y_{4i} = -(l_{y4i} + e_y) = -(\beta_{4i} + \eta_y) l_y \tag{6d}$$

Where:

i…Location of springs and shock absorbers in the quadrant j (where $i = 1, 2, \ldots, n$; $j = 1, \ldots, 4$);

$a_{ji} = l_{xji} / l_x$;

$\beta_{ji} = l_{yji} / l_y$;

$\eta_x = e_x / l_x$;

$\eta_y = e_y / l_y$, and l_x;

l_y…Reference values characteristic for a given case, e.g., plate dimensions. This determines the geometry of the spring and shock absorber mounting locations.

For the selected coordinate system x, y, z with the origin at the center of gravity *T* of the body, the displacement at the center of gravity is $u_0 = u_T = 0$, $v_0 = v_T = 0$ and $w_0 = w_T = z$. The vertical displacement

at the location of individual springs and shock absorbers $i = 1$ to n in individual quadrants $j = 1, ..., 4$ is given by:

$$w_{ji} = z + x_{ji}\varphi_y - y_{ji}\varphi_x - h_{ji}(t) \tag{7}$$

Then, the displacement of any point on the plate is given by the sum of the displacement of the center of gravity w_T, i.e., the displacement caused by rotation around the x-axis ($-y_{ji}\varphi_x$), the displacement caused by rotation around the y-axis ($x_{ji}\varphi_y$), and the change in the inertial basis $h_{ji}(t)$.

Kinetic energy is obtained by modifying relation (2):

$$E_k = \frac{1}{2}\left[m\dot{z}^2 + J_x\dot{\varphi}_x^2 + J_y\dot{\varphi}_y^2 - 2D_{xy}\dot{\varphi}_x\dot{\varphi}_y\right] \tag{8}$$

And the potential energy for $\varkappa = 0$ is obtained by modifying (3) with regard to (7):

$$E_p = \frac{1}{2}\sum_{j=1}^{4}\sum_{i=1}^{n_j} w_{ji}^2 k_{ji} = \frac{1}{2}\sum_{j=1}^{4}\sum_{i=1}^{n_j} (z + x_{ji}\varphi_y - y_{ji}\varphi_x - h_{ji}(t))^2 k_{ji} \tag{9}$$

By substituting (7) into (9) and performing the appropriate derivations, we obtain the relations for the kinematic excitation function:

$$H_x(t) = -\sum_{j=1}^{4} k_j \sum_{j=1}^{n_j} \gamma_{ji} h_{ji}(t) y_{ji}$$

$$H_y(t) = -\sum_{j=1}^{4} k_j \sum_{j=1}^{n_j} \gamma_{ji} h_{ji}(t) x_{ji} \tag{10}$$

$$H_z(t) = \sum_{j=1}^{4} k_j \sum_{j=1}^{n_j} \gamma_{ji} h_{ji}(t)$$

Where:

k_j…Average stiffness in the j-th quadrant ($k_j = \sum_{i=1}^{n_j} \frac{k_{ji}}{n_j}$, $\gamma_{ji} = k_{ji}/v$),

n_j…Number of springs in the j-th quadrant.

The kinematic excitation function $H_j(t)$ can be obtained from the generalized excitation force Q_j (right-hand side of equation (1)).

In equation (1), for a spatially flexible body, it is clear that $\partial E_k/\partial q_j = 0$ and the components of the excitation function $F_j(t)$ are given by the sum of the excitation force Q_j and the components of the kinematic excitation $f_{Kj}(h(t))$ and dissipative excitation $f_{Dj}(\dot{h}(t))$.

Then, the excitation function $F_j(t)$ can be determined under the assumption of a single external potential force (weight of body $Q_j = G$) from the relationships:

$$F_z(t) = G + f_{Kz}(h(t)) + f_{Dz}(h(t))$$

$$F_x(t) = f_{Kx}(h(t)) + f_{Dx}(h(t)) \tag{11}$$

$$F_y(t) = f_{Ky}(h(t)) + f_{Dy}(h(t))$$

The above relationships characterize the general equations of motion of forced vibration of a spatially flexibly mounted body (rigid plate) as a system with 3 degrees of freedom with general excitation load, considering energy dissipation and the influence of mounting asymmetry and mass distribution.

The article deals with solving vertical vibrations of a spatial model of a mechanical system with complete asymmetry, both structural and operational. The solution of this system is based on Lagrange's equation of the second kind (1). Based on these assumptions, simple 3D models of a mechanical system were designed that meet the required assumptions and allow for changes in asymmetry (structural, loading, and sus-pension). These models are suitable for both analytical and experimental solutions.

3 Methodology

A general 3D model of a mechanical system was designed to analyze various cases of asymmetry in vertical vibration (Figure 3).

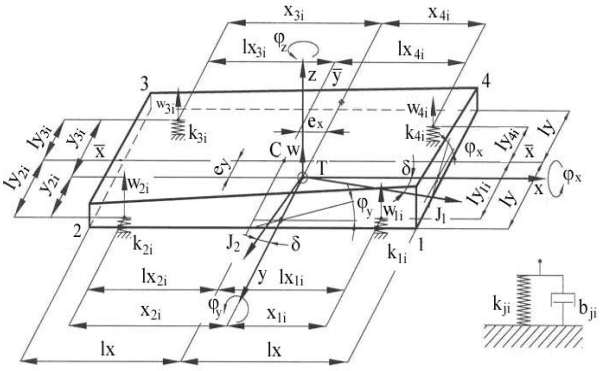


Fig. 3 *General solution model*

Where:

A…Any point;

C…Geometric center;

T…Position of the center of gravity;

$V_{1,2}$…Counterweights;

ZD…Base plate;

e_x, e_y…Eccentricity of the center of gravity;

l_x, l_y…Distance of mounting (springs, shock absorbers) from the center of gravity;

x_A, y_A…Coordinates of point A, 1, 2, 3, 4 – mounting position (spring attachment).

For the purpose of solving the above model, an x, y, z coordinate system was introduced with its origin at the center of gravity T. The z coordinate of the center of gravity T does not change relative to the position of the coordinate at the geometric center C ($z_C = z_T = 0$). This model allows the load distribution to be changed by changing the position of the counterweights, as well as allowing the stiffness of the mounting to be changed by changing the position of the springs and shock absorbers, including their stiffness.

The criterion for comparing the individual variants was the time course of the vertical displacement of the center of gravity, individual selected points, and angles of rotation around the x and y axes. The vertical displacement of any point A is given by the relationship:

$$w_A(t) = w_T(t) + x_A\varphi_y(t) - y_A\varphi_x(t) \tag{12}$$

Where:

w_A…Deviation of point A;

w_T…Deviation of the center of gravity;

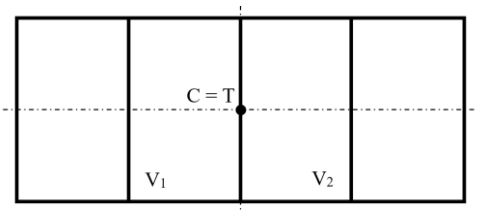
x_A, y_A…Coordinates of point A;

φ_x, φ_y…Rotation of the body around the x and y axes;

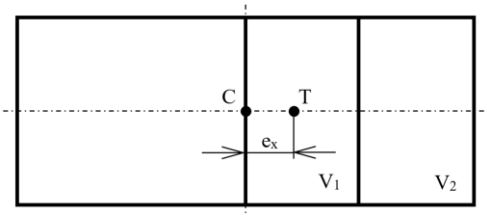
t…Time.

Based on the general model of a mechanical system (Figure 3), seven variants of the model layout were designed, which allow for solving symmetric and asymmetric models of mass distribution and symmetric and asymmetric excitation (Table 2).

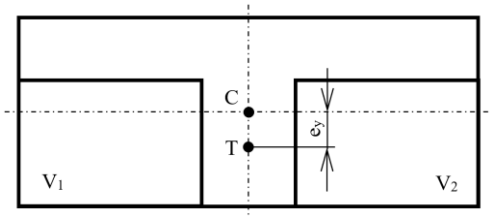
Tab. 2 *Arrangement of models of the mechanical system under consideration*



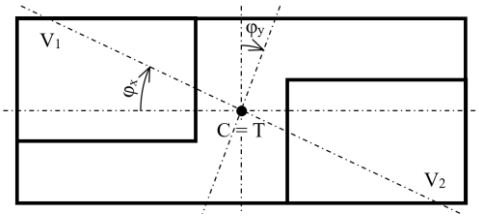
Symmetrical arrangement – variant I



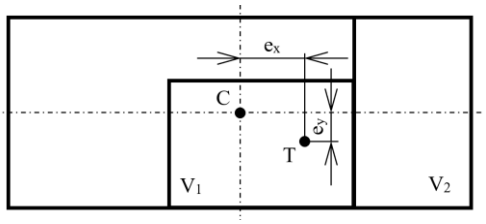
Asymmetrical arrangement – variant II



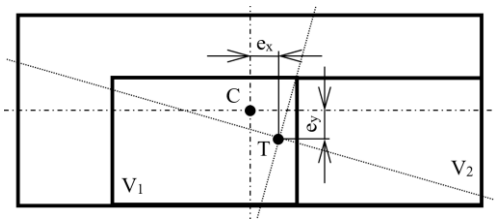
Asymmetrical arrangement – variant III



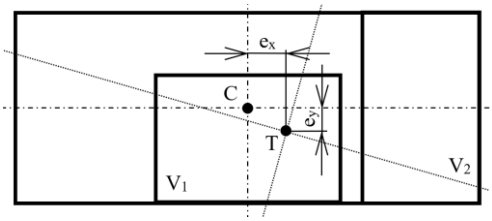
Asymmetrical arrangement – variant IV



Asymmetrical arrangement – variant V



Asymmetrical arrangement – variant VI



Asymmetrical arrangement – variant VII

For the above-mentioned variants of the mechanical system arrangement, kinematic excitation with a unit step (Heaviside function) was proposed. The proposed solution allows for both symmetric and asymmetric kinematic excitation (Table 3).

Tab. 3 *Cases of kinematic excitation of a mechanical system*

Labeling	Changing the position of points		Note
S	$h_1 = h_2 = h_3 = h_4 = 5$ mm		symmetrically excited system
1	$h_1 = h_4 = 5$ mm	$h_3 = h_2 = 0$	asymmetrically excited system
2	$h_1 = h_2 = 0$	$h_3 = h_4 = 5$ mm	asymmetrically excited system
3	$h_1 = 5$ mm	$h_2 = h_3 = h_4 = 0$	asymmetrically excited system
4	$h_1 = h_2 = h_4 = 5$ mm	$h_3 = 0$	asymmetrically excited system
5	$h_1 = h_3 = 5$ mm	$h_2 = h_4 = 0$	asymmetrically excited system

Where:
h_i…Change of point position $i = 1, 2, 3, 4$.

4 Solution

Selected cases of vertical vibration of the proposed mechanical system were analytically solved on a simple model (Figure 3). Kinematic excitation was induced by a unit step with a value of $h = 1$. Then, the vertical displacements at individual points 1, 2, 3, 4 – see Figure 3 – can be expressed by the following relationships:

$$\begin{aligned} w_1(t) &= w_T(t) + (l_{x1} - e_x)\varphi_y(t) - (l_{y1} - e_y)\varphi_x(t) - h_1(t) \\ w_2(t) &= w_T(t) - (l_{x2} + e_x)\varphi_y(t) - (l_{y2} - e_y)\varphi_x(t) - h_2(t) \\ w_3(t) &= w_T(t) - (l_{x3} + e_x)\varphi_y(t) + (l_{y3} + e_y)\varphi_x(t) - h_3(t) \\ w_4(t) &= w_T(t) + (l_{x4} - e_x)\varphi_y(t) + (l_{y4} + e_y)\varphi_x(t) - h_4(t) \end{aligned} \tag{13}$$

General for point A:

$$w_A(t) = w_T(t) - (x_A + e_x)\varphi_y(t) + (y_A + e_y)\varphi_x(t) - h_A(t) \tag{14}$$

Where:
e_x, e_y…Eccentricity of the center of gravity T from the geometric center C in the x and y axes;
h_i…Deviation of points 1, 2, 3, 4, A;
l_{xi}, l_{yi},…Distance of points from the center of gravity on the x and y axes;
w_T…Center of gravity deviation;
w_1, w_2, w_3, w_4, w_A…Deviation of points 1, 2, 3, 4, A;
φ_x, φ_y…Angles of rotation of the body around the x and y axes.

Cases without damping were solved, i.e., the term characterizing damping was omitted from the equations of motion. The equation of motion can then be written in the form:

$$\mathrm{M}\ddot{q}_j(t) + \mathrm{K}q_j(t) = \mathrm{F}_j(t) \quad \text{for } j = 1, 2, 3 \tag{15}$$

Where the generalized coordinate $q_j = \left[w_T(t), \varphi_x(t), \varphi_y(t)\right]^T$.

The vertical component of force $\Phi_i(t)$ at the i-th support point is given by the following equation for a system without damping:

$$\Phi_i(t) = k_i h_i(t) \tag{16}$$

Where:
k_i…Spring stiffness constant at point i,
$h_i(t)$…Change in the position of the center of gravity due to the action of internal or external forces.

Then the components of the generalized function $F_j(t)$ are given by the relations:

$$F_1(t) = m^{-1} \sum_{m=1}^{4} k_m h_m(t)$$

$$F_2(t) = J_x^{-1} \left[- \sum_{m=1}^{2} k_m h_m(t) l_{ym} + \sum_{m=3}^{4} k_m h_m(t) l_{ym} + e_y \sum_{m=1}^{4} k_m h_m(t) \right] \quad (17)$$

$$F_3(t) = J_y^{-1} \left[\sum_{m=1,4} k_m h_m(t) l_{xm} - \sum_{m=2,3} k_m h_m(t) l_{xm} + e_x \sum_{m=1}^{4} k_m h_m(t) \right]$$

Where:

F_1…Component of generalized force (physically, this is a component of vertical acceleration),

F_2…Component of the generalized function (physically, this is the component of angular acceleration relative to the x-axis),

F_3…Component of the generalized function (physically, the component of angular acceleration relative to the y-axis),

l_{xm}, l_{ym} (for $m = 1, 2, 3, 4$)…Distance of the (flexible viscoelastic damping) from the $\bar{x}$ and $\bar{y}$ axes of geometric symmetry with the origin at point C.

5 Results and discussion

The analytical calculation was performed for an undamped mechanical system. The calculation considered the kinematic excitation of the system by a unit step, which caused the spring to be displaced by one unit (unit step) at its mounting point. The analytical solution was performed for the following parameters of the mechanical system.

Tab. 4 *Geometrical and Mechanical Parameters of the Analysed Mechanical System*

Springs	mean stiffness value	k = 16.49 ± 0.12 Nmm^{-1}
Plate	length	l_x = 0.690 m
	width	l_y = 0.280 m
	height (thickness)	l_z = 0.029 m
	weight	m = 43.702 kg
Balancing weights ($V_1 = V_2$)	length	l_{x1} = 0.1725 m
	width	l_{y1} = 0.280 m
	height	l_{z1} = 0.058 m
	weight of 1 piece	m_1 = 21.851 kg
Spring placement (arranged symmetrically to the geometric axes):		
Spacing in the axis	x	l_a = 0.67 m
	y	l_b = 0.26 m

Asymmetrical weight distribution was achieved by placing counterweights at different positions on the base plate, thereby shifting the center of gravity of the system T relative to the geometric center of the base plate C ($T \neq C$, for a symmetrical arrangement $T = C$). The average stiffness of the springs was determined experimentally (springs with the closest matching stiffness values were selected).

For individual types of kinematic excitation of the system, the following were determined analytically:

- $w_T(t)$, $\varphi_x(t)$, $\varphi_y(t)$ – vertical displacement of the center of gravity and rotation around axes passing through the center of gravity,
- $w_1(t)$, $w_2(t)$, $w_3(t)$, $w_4(t)$ – vertical displacement of points 1 to 4.

For each solution variant according to Table 2, the mass moments of inertia of the mechanical system were determined from the partial moments listed in Table 5. The conversion was performed using a modified version of Steiner's theorem.

Tab. 5 *Moment of inertia of individual bodies*

Body	Mass moment of inertia [kg·mm²]		
	J_x	J_y	J_z
Base plate (ZD)	288 582.52	1 736 939.60	2 019 396.60
Balancing weights (V_1)	148 885.43	60 309.22	196 443.52
Balancing weights (V_2)	148 885.43	60 309.22	196 443.52

The mechanical system model was mounted on four reinforced springs (cylindrical), whose position from the axis passing through the geometric center was (Table 6).

Tab. 6 *Position of springs on a mechanical system relative to the geometric center C*

Spring	Coordinate *x* [mm]	Coordinate *y* [mm]
1	335	-130
2	-335	-130
3	-335	130
4	335	130

The vertical shifts of individual points were determined according to the relationships (13). In individual solutions, the following notation is chosen for the vertical movement of individual points:

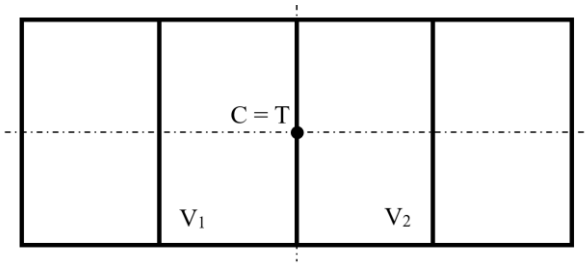
w_{Xij}
- cases of kinematic excitation (S, 1, 2, 3, 4, 5)
- storage location (springs, shock absorbers, etc. 1, 2, 3, 4)
- model arrangement variant (I, II, III, IV, V, VI, VII)

The numerical calculation was performed in MATLAB, and the graphical outputs are from MS Excel.

Selected results characterizing the vertical vibration of the given mechanical system arrangement from Table 2 are listed below.

5.1 Variant I

This is a symmetrical system, characterized as follows:



Mass moment of inertia [kg·m²]		
J_x	J_y	J_z
0.62770093	2.22300750	2.73838550
Eccentricity of the center of gravity [mm]		
e_x	e_y	
0	0	

Time course of the deflection of the center of gravity of a flexibly mounted plate $z_T(t) \equiv w_T(t)$. At other points, i.e., at the points where the springs are mounted to the base plate (ZD), this course is represented by the corresponding curves. The resulting graphs show the course of deflections and rotations along the axes in the time interval $t = 2.72 \div 3.16$ s.

5.1.1 Variant I, symmetrical excitation

This symmetrically arranged mechanical system is excited symmetrically (S – see Table 3), i.e., points 1 to 4 move simultaneously by the same value in the downward direction.

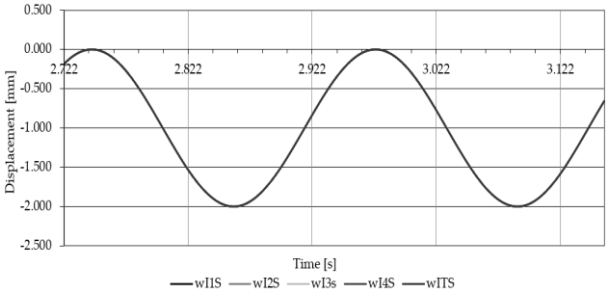


Fig. 4 *Vertical vibrations of points of a symmetric mechanical system – variant I, symmetrically excited*

This movement causes harmonic vibration of the entire system (Figure 4). The shape of the vibrations is sinusoidal. The maximum deflection value in the z-axis is -0.0002 mm and the minimum value is -1.9997 mm. The amplitude of the vibration is 1.9995 mm. With this excitation, there is no rotation of the axes of symmetry of the system passing through the center of gravity.

5.1.2 Variant I, asymmetric excitation 1

This symmetrically arranged mechanical system is excited asymmetrically according to case 1 (Table 2). The vertical vibrations of this system are shown on Figure 5.

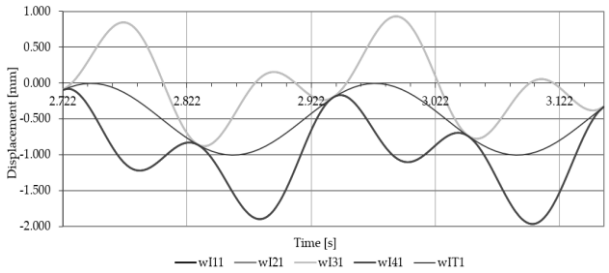


Fig. 5 *Vertical vibrations of points of a symmetric mechanical system – variant I, asymmetrically excited, case 1*

From the movement of individual points, it is clear that the curves of points w_{I11} and w_{I31} overlap, with a maximum deviation of 0.9309 mm at 2.9888 s and a minimum deviation of -0.8824 mm at 2.8364 s. Similarly, the curves of the movement of points w_{I21} and w_{I41} overlap, with a maximum deviation of -0.0786 mm at 2.7276 s and a minimum deviation of -1.9568 mm at 3.0978 s. The movement of the center of gravity is sinusoidal, with a minimum deviation of the center of gravity T of -0.9999 mm at time 2.8582 s and a maximum deviation of -0.0001 mm at time 2.7439 s. The minimum deviation value in this case is smaller than in the previous case and is almost half as much. The situation is similar for the maximum deflection value, which in this case is half that of the previous case. This can be attributed to the fact that in this case, points 1 and 4 have fallen, the plate is tilted after the jump and oscillates in this tilted plane.

Figure 6 shows the rotation of the mechanical system around the y-axis by an angle φ_y. The rotation is sinusoidal, with a maximum angle of rotation of φ_y = -2.9851° and a minimum angle of rotation of φ_y = 0°. The system does not rotate around the x-axis.

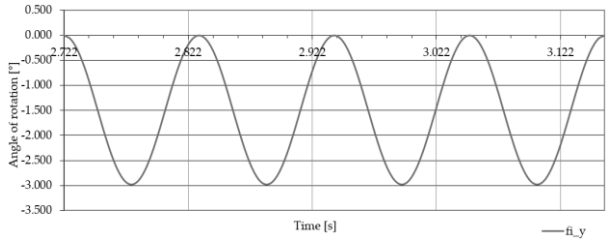


Fig. 6 *Rotation around the y-axis of a symmetrical mechanical system – variant I, asymmetrically excited, case 1*

5.1.3 Variant I, asymmetric excitation 2

This symmetrically arranged mechanical system is excited asymmetrically according to case 2 (Table 3). In this case, the mechanical system tilts around the x-axis; the vertical vibration curve is shown on Figure 7.

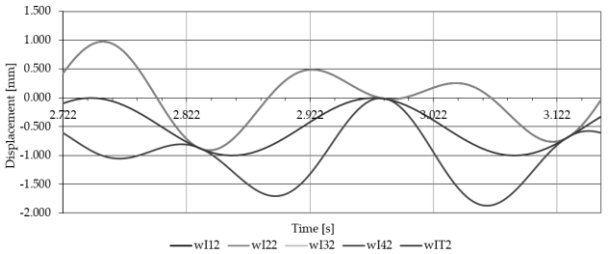


Fig. 7 *Vertical vibrations of points of a symmetric mechanical system – variant I, asymmetrically excited, case 2*

The movements of points w_{I12} and w_{I22} are identical, which is also evident from the deflection curves in the z-axis – Figure 7. The deflection reaches a minimum value of -0.9100 mm at time t = 2.8419 s. The maximum deflection value of these points is 0.0975 mm at time t = 2.7548 s. After that, the deflections are smaller and then repetition occurs. The movement of points w_{I32} and w_{I42} is the same, the maximum deflection is reached at -0.0100 mm at time t = 2.9780 s and the minimum deflection is -1.8741 mm at time t = 3.0651 s. The deflection of the center of gravity is periodic, sinusoidal, with a maximum value of -0.0001 mm at time t = 2.7439 s and a minimum deflection value of -0.9998 mm at time t = 2.8582 s, after which the movement repeats.

After springs 3 and 4 are compressed, the plate tilts around the x-axis and oscillates around this axis (Figure 8). The mean value of the plate's rotation is -3.8454°, around which the plate rotates. The minimum rotation value is φ_x = -7.6922° and the maximum value is φ_x = -0.0014°. The curve is again sinusoidal. The mechanical system around the y-axis does not oscillate.

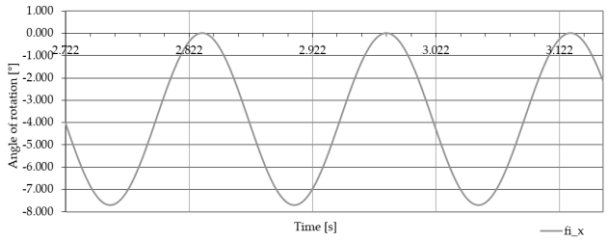


Fig. 8 *Rotation around the x-axis of a symmetrical mechanical system – variant I, asymmetrically excited, case 2*

5.1.4 Variant I, asymmetric excitation 4

This symmetrically arranged mechanical system is excited asymmetrically according to case 4 (Table 3). In this case, the mechanical system tilts around the x and y axes; the vertical vibration curve is shown on Figure 9.

As in all cases of oscillation in a symmetrical mechanical system, the movement of the center of gravity is sinusoidal and periodically repeating. The center of gravity oscillates around a mean value of -0.7498 mm, with a period of approximately 0.2286 s.

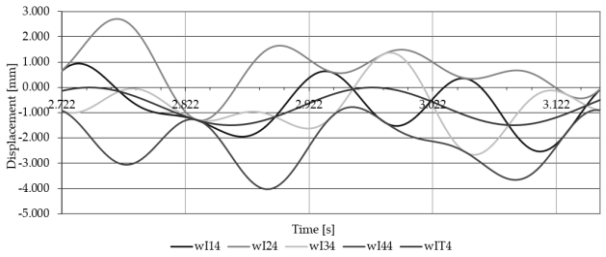


Fig. 9 *Vertical vibrations of points of a symmetric mechanical system – variant I, asymmetrically excited, case 4*

Point w_{I14} reaches a maximum deflection of 0.9318 mm at time t = 2.7384 s and a minimum deflection of -2.5323 mm at time t = 3.1087 s. Between these points, it reaches 2× local minima and 2× local maxima. Point w_{I24} reaches a maximum vertical deflection of 2.6996 mm at time t = 2.7657 s and a minimum deflection of -1.3152 mm at time t = 2.8364 s. At point w_{I34}, the maximum deflection in the z-axis direction is 1.3647 mm at time 2.9888 s, and the minimum deflection at this point is -2.6636 mm at time t = 3.0542 s. At point w_{I44}, the maximum deviation is -0.7910 mm at time t = 2.9562 s and the minimum deviation is -4.0188 mm at time t = 2.8909 s.

The excited system rotates around both axes (x, y). The rotation curves of the symmetrical mechanical system excited asymmetrically are shown on Figure 10. The curves of both rotations are sinusoidal with different frequencies. The frequency of rotation around the y-axis is higher than around the x-axis, similar to the previous case. The mean value of the rotation amplitude around the y-axis is 2.2388°, and around the x-axis it is 5.7681°.

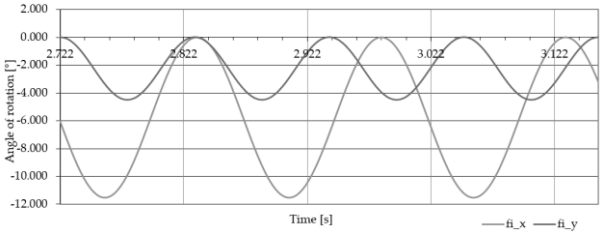


Fig. 10 *Rotation around the x and y axes of a symmetrical mechanical system – variant I, asymmetrically excited, case 4*

5.1.5 Variant I, point 1 movement – all excitation variants

The vertical displacement of point 1 (Figure 3) of the symmetrical mechanical system for all cases of kinematic excitation (Table 3) is shown on Figure 11.

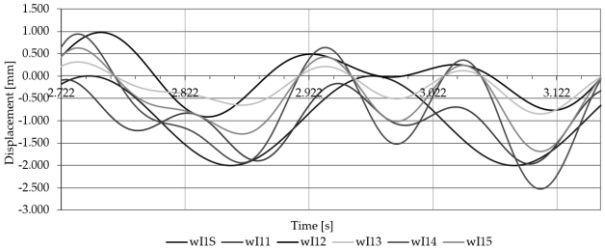


Fig. 11 *Vertical vibrations of point 1 of a symmetric mechanical system – variant I, all excitation cases*

The characteristic values of vertical vibration at point 1 for all excitation cases are given in Table 7.

Tab. 7 *Characteristic values of vertical vibration of point 1 for all excitation cases*

Cases of excitation	Labeling	Deviation max. [mm]	in time [s]	min. [mm]	in time [s]
S	w_{I1S}	-0.0002	-	-1.9998	-
1	w_{I11}	-0.0786	2.7276	-1.9568	3.0978
2	w_{I12}	0.9752	2.7548	-0.9100	2.8419
3	w_{I13}	0.3106	2.7384	-0.8441	3.0542
4	w_{I14}	0.9318	2.7384	-2.5323	3.1087
5	w_{I15}	0.6212	2.7384	-1.6882	3.1087

Table 7 shows that in excitation cases 3, 4, and 5, the maximum vertical deflection extremes occur at the same time, and in excitation cases 4 and 5, the minimum deflection extremes again occur at the same time. In all cases of kinematic excitation (Table 6), the vertical movement of the center of gravity in a given time interval was sinusoidal and is represented by the movement of all points (1, 2, 3, 4) during symmetrical ex-citation; time is not specified in this case.

5.1.6 Variant I, point 4 movement – all excitation variants

From the perspective of the vertical vibration curves of a symmetrical mechanical system for all excitation variants, point 4 is interesting (Figure 12).

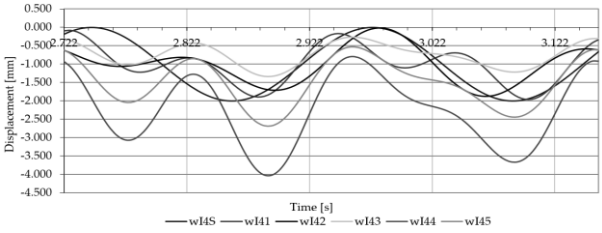


Fig. 12 *Vertical vibrations of point 4 of a symmetric mechanical system – variant I, all excitation cases*

Table 8 shows the characteristic values of vertical deflections of point 4.

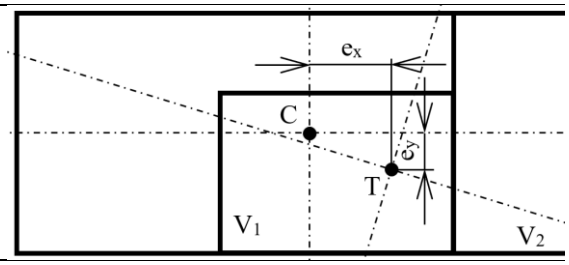
Tab. 8 *Characteristic values of vertical vibration of point 4 for all excitation cases*

Cases of excitation	Labeling	Deviation max. [mm]	Deviation in time [s]	Deviation min. [mm]	Deviation in time [s]
S	w_{14S}	-0.0002	-	-1.9998	-
1	w_{141}	-0.0786	2.7276	-1.9568	3.0978
2	w_{142}	-0.0100	2.9780	-1.8741	3.0651
3	w_{143}	-0.2637	2.9562	-1.3596	2.8909
4	w_{144}	-0.7910	2.9562	-4.0188	2.8909
5	w_{145}	-0.5274	2.9562	-2.6792	2.8909

From Table 8, it follows that the extreme values of the deviations of point 4 are at the same times (not values) for excitation cases 3, 4, and 5.

5.2 Variant V

Given the scope of this article, we present some results for variant 5, i.e., the asymmetrically arranged variant for selected cases of asymmetric kinematic excitation. This variant is characterized as follows.



Mass moment of inertia [kg·m²]		
J_x	J_y	J_z
0.58647139	3.01013420	3.48328270
Eccentricity of the center of gravity [mm]		
e_x		e_y
78.813		13.438

The time course of vertical vibrations of points and angles of rotation of the mechanical system around the x and y axes is shown in the time interval t = 3.173÷ 3.681 s.

5.2.1 Variant V, asymmetric excitation 2

The course of vertical vibration of an asymmetrical mechanical system, which is asymmetrically excited according to case 2 (Table 3), at points 1 to 4 and the center of gravity is shown on Figure 13.

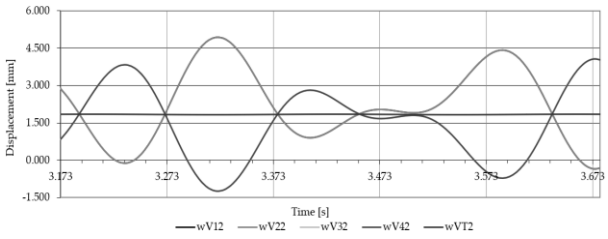


Fig. 13 *Vertical vibrations of points of an asymmetrical mechanical system – variant V, asymmetrically excited, case 2*

The characteristic values of vertical movement of points are given in Table 9.

Tab. 9 *Characteristic values of vertical vibration of points and for excitation case 2*

Labeling of point	Deviation max. [mm]	Deviation in time [s]	Deviation min. [mm]	Deviation in time [s]	Labeling of point	Deviation max. [mm]	Deviation in time [s]	Deviation min. [mm]	Deviation in time [s]
w_{V12}	4.9340	3.3190	-0.3533	3.6744	w_{V42}	4.0722	3.6744	-1.2288	3.3190
w_{V22}	4.9340	3.3190	-0.3533	3.6744	w_{VT2}	1.8598	3.6617	1.8526	3.5475
w_{V32}	4.0722	3.6744	-1.2288	3.3190	-	-	-	-	-

From Table 9. it follows that points 1, 2, and 3, 4 oscillate identically with the same amplitude and frequency. Points 3 and 4 have the maximum deflection of 5.3010 mm. Points 1 and 2 have a smaller deflection of 5.2873 mm. The difference between the maximum deflection of both pairs of points is negligible. The course of both curves shows that the curves are mirror images of each other in terms of time, but differ in amplitude (slightly). The center of gravity of the mechanical system performs a sinusoidal motion with an amplitude of 0.0360 mm, which means that the center of gravity hardly changes its position.

The rotation of this mechanical system around the x-axis is shown on Figure 14.

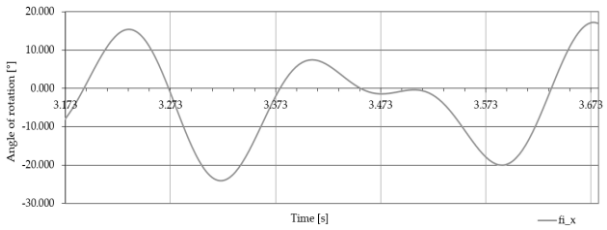


Fig. 14 *Rotation around the x-axis of an asymmetrical mechanical system, variant V, asymmetrically excited, case 2*

The rotation around the x-axis is similar to the vertical movement of points 3 and 4. The maximum rotation angle is φ_x = 17.2529°, and the minimum rotation angle is φ_x = -24.0259°. The total rotation range is 41.2788°. There is no rotation of the system around the *y*-axis.

The course of these vertical vibrations of all monitored points is shown on Figure 15.

The characteristic values of vertical vibration of points are given in Table 10.

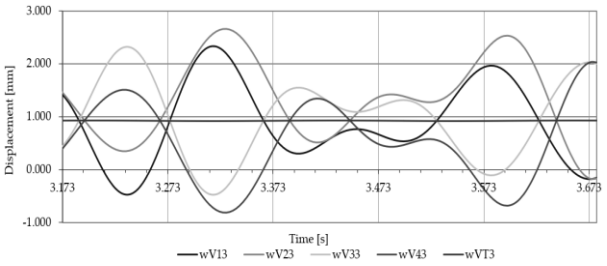


Fig. 15 *Vertical vibrations of points of an asymmetric mechanical system – variant V, asymmetrically excited, case 3*

Tab. 10 *Characteristic values of vertical vibration of points and for excitation case 3*

Labeling of point	Deviation max. [mm]	Deviation in time [s]	Deviation min. [mm]	Deviation in time [s]	Labeling of point	Deviation max. [mm]	Deviation in time [s]	Deviation min. [mm]	Deviation in time [s]
w_{V13}	2.3231	3.3190	-0.4635	3.2365	w_{V43}	2.0385	3.6744	-0.8052	3.3253
w_{V23}	2.6578	3.3253	-0.1790	3.6744	w_{VT3}	0.9299	3.6617	0.9263	3.5475
w_{V33}	2.3217	3.2365	-0.4705	3.3190	-	-	-	-	-

Points 1 and 3 reach local extremes at the same times, with point 1 reaching its maximum at the same time that point 3 reaches its minimum, and vice versa. Points 2 and 4 moves in the same way. Point 4 reaches the largest deflection of 2.8437 mm, followed by point 2 with a deflection of 2.8368 mm, while points 1 and 3 do not reach a deflection of even 2.8 mm. The center of gravity again performs a sinusoidal harmonic motion with a negligible deflection of 0.0036 mm, i.e., with an amplitude of 0.0018 mm.

The course of the angles of rotation around the x and y axes is shown on Figure 16.

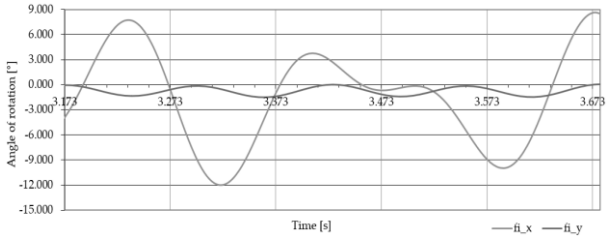


Fig. 16 *Rotation around the x and y axes of an asymmetrical mechanical system, variant V, asymmetrically excited, case 3*

It is evident from the arrangement that the angle of rotation around the *y*-axis is very small, with an oscillation of 1.5113°. The rotation around the *x*-axis is greater, at 20.6413°.

5.2.2 Variant V, asymmetric excitation 4

The vertical vibrations of the points according to this excitation case are shown on Figure 17.

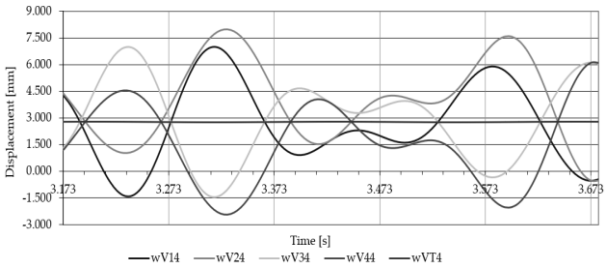


Fig. 17 *Vertical vibrations of points of an asymmetric mechanical system – variant V, asymmetrically excited, case 4*

An overview of local extremes of deviations of individual points is given in Table 11.

Tab. 11 *Characteristic values of vertical vibration of points and for excitation case 3*

Labeling of point	Deviation max. [mm]	Deviation in time [s]	Deviation min. [mm]	Deviation in time [s]	Labeling of point	Deviation max. [mm]	Deviation in time [s]	Deviation min. [mm]	Deviation in time [s]
w_{V14}	6.9692	3.3190	-1.3904	3.2365	w_{V44}	6.1156	3.6744	-2.4155	3.3253
w_{V24}	7.9735	3.3253	-0.5371	3.6744	w_{VT4}	2.7897	3.6617	2.7788	3.5475
w_{V34}	6.9652	3.2365	-1.4115	3.3190	-	-	-	-	-

From the above overview, it follows that the movement of the center of gravity is again sinusoidal with a maximum oscillation of 0.0109 mm, i.e., with an amplitude of approximately 0.00545 mm. The largest amplitude is achieved by point 4, namely 8.5311 mm, followed by point 2 with an amplitude

of 8.5106 mm. Both points again reach local extremes at the same times, with point 4 reaching its maximum value when point 2 is at its minimum value, and vice versa. Points 1 and 3 oscillate in the same way, but their maximum amplitude is smaller, reaching 8.3767 mm (point 3) and 8.3596 mm (point 1).

The angles of rotation of this mechanical system around the *x* and *y* axes are shown on Figure 18.

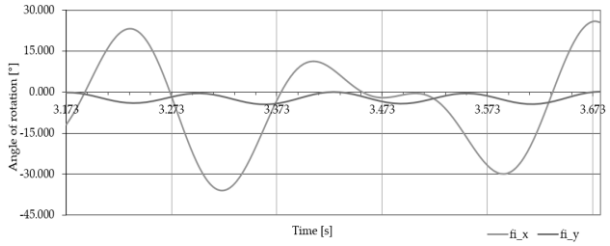


Fig. 18 *Rotation around the x and y axes of an asymmetrical mechanical system, variant V, asymmetrically excited, case 4*

The rotation of the system around the *y*-axis is minimal, and the total deviation is 4.5639°. The system deviates around the mean value of -2.1622°. Around the *x*-axis, the rotation is significantly greater, and the maximum deviation is 61.9181°. The maximum amplitude is 30.959°, and the system rotates around the *x*-axis around a mean value of -10.160°.

5.2.4 Variant V, point 2 movement – all excitation variants

Figure 19 graphically shows the vibration of point 2 according to this variant for all excitation cases.

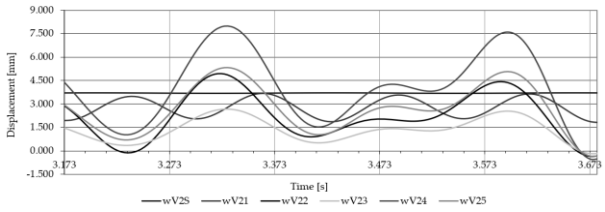


Fig. 19 *Vertical vibrations of point 2 of an asymmetrical mechanical system – variant V, for all excitation cases*

During symmetrical excitation, all points, including the center of gravity (w_{V2S}), move identically along a sine wave with an amplitude of 0.0144 mm. It is important to note that we are observing the time when the oscillation has already stabilized after excitation.

Excitation case 1 is again characterized by a sine wave with variable amplitude, or variable deviation, which is determined both by the distribution of masses and by the method of excitation. In excitation cases 3, 4, and 5, point 2 reaches local extremes at the same times, both minimum and maximum. However, the deviation values are different (Table 12).

Tab. 12 *Characteristic values of vertical vibration of point 2 for all excitation cases*

Cases of excitation	Labeling	Deviation			
		max. [mm]	in time [s]	min. [mm]	in time [s]
S	w_{V2S}	3.7195	-	3.7051	-
1	w_{V21}	3.6838	3.3634	1.8099	3.6807
2	w_{V22}	4.9340	3.3190	-0.3533	3.6744
3	w_{V23}	2.6578	3.3253	-0.1790	3.6744
4	w_{V24}	7.9735	3.3253	-0.5371	3.6744
5	w_{V25}	5.3157	3.3253	-0.3571	3.6744

6 Conclusion

The article presents some results of the analytical solution of undamped vertical vibration of a model mechanical system. The solution methodology is briefly described, a simple 3D model of this system is proposed, and the relevant analytical calculations are performed. The solution is based on Lagrange's equation of the second kind, which has been suitably modified for these cases. The article presents the vertical deflection curves of an undamped mechanical system with different mass distributions and different kinematic excitations.

The results confirm the assumption that the distribution of mass in a given system has a decisive influence on the vibration of that system, and that the method of kinematic excitation also has a significant influence. The results clearly show that when designing mechanical systems (machines), it is particularly important to solve symmetrical systems with symmetrical excitation. Therefore, every effort should be made to achieve this symmetry as closely as possible in every machine design, both in terms of mass distribution and kinematic excitation. If the machine design deviates from this symmetry (which is almost always the case), mechanical intervention is necessary to limit the effects of vertical vibration and thus stabilize the operation of the system (machine). The solution to vibration is particularly important in machine tools, where vibration is a decisive factor for their accuracy. Since most mechanical systems (machines) contain rotating members with unbalanced masses, it is necessary to focus on their balancing.

The asymmetrical distribution of mass is particularly evident in vehicles, where it is inherently impossible to achieve structural and load symmetry. Due to the uneven-ness of the road surface, the vibration of these vehicles is always asymmetrical. In extreme cases, vibration can even cause the wheel or wheels to lose contact with the road surface, which can lead to an accident.

From the point of view of the vibration of a mechanical system, or rather its impact on the surrounding area, it is very important to use flexible mountings. Mountings with linear damping characteristics are most commonly used. However, non-linear damping (springs) is also used in vehicles and machines.

In the next section, we will focus on solving vertical vibrations of a damped mechanical system (machine).

Acknowledgement

Supported by the OPST Project Green Energy Technologies Centre of UJEP, Reg. number CZ.10.02.01/00/24_061/0000462.

References

[1] SOUKUP, J., VOLEK, J., et al. (2008). *Vibrations of mechanical systems - vehicles. Analysis of the influence of asymmetry. (Kmitání mechanických soustav - vozidel. Analýza vlivu nesymetrie)*, 1st ed., Acta Universitatis UJEP in Ústí n. L, Ústí n. L., pp. 269, ISBN 978-80-7414-020-4

[2] PUST, L., LADA, M. (1965). *Flexible machine storage. (Pružné ukládání strojů),* SNTL Praha, 1965, pp. 190

[3] ZEMAN, V., HLAVAC, Z. (2004). *Vibration of mechanical systems. (Kmitání mechanických soustav),* ZČU Plzeň, Fakulta aplikovaných věd, Plzeň, 2004

[4] AMER, M.; WALLASCHEK, J.; SEUME, J.R. (2022). Operational Modal Analysis of an Axial Compressor Rotor and Casing System for the Online Identification of a Digital Twin. *Appl. Mech. 2022*, No. 3, pp. 244-258. https://doi.org/10.3390/applmech3010017

[5] AWREJCEWICZ, J. (2008). Mechanika, Wydawnictwa Naukowo-Techniczne, *Fundacja Książka "Naukowo-Techniczna",* Warszawa, 2008, p. 580, ISBN 978-83-924382-3-6

[6] SHEN, C., LU, C. (2024). A Method to Obtain Frequency Response Functions of Operating Mechanical Systems Based on Experimental Modal Analysis and Operational Modal Analysis. *Machines* 2024, 12, 516. https://doi.org/10.3390/machines12080516

[7] KASHANI, M., HASHEMI, S.M. (2022) On the Free Vibration and the Buckling Analysis of Laminated Composite Beams Subjected to Axial Force and End Moment: A Dynamic Finite Element Analysis. *Appl. Mech. 2022*, 3, 210-226. https://doi.org/10.3390/applmech3010015

[8] NAKANO, Y., KISHI, T., TAKAHARA, H. (2021). Experimental Study on Application of Tuned Mass Dampers for Chatter in Turning of a Thin-Walled Cylinder. *Appl. Sci.* 2021, 11, 12070. https://doi.org/10.3390/app112412070

[9] BRAUN, S., EWINS, D., RAO, S. SW. (2002). *Encyklopedia of Vibration.* New York: Academic Press, New York, 2002

[10] DE SILVA, CLARENCE W. (ed.) (2005). *Vibration and shock handbook*. Mechanical engineering series. Boca Raton: Taylor & Francis, 2005. ISBN 0-8493-1580-8.

[11] MATANIA, O., BACHAR, L., BECHHOEFER, E., BORTMAN, J. (2024). Signal Processing for the Condition-Based Maintenance of Rotating Machines via Vibration Analysis: A Tutorial. *Sensors* 2024, 24, 454. https://doi.org/10.3390/s24020454

[12] STEJSKAL, V., OKROUHLÍK, M. (2002). *Kmitání s Matlabem. (Vibration with Matlab)*, ČVUT Praha, 376 str., Praha, 2002, ISBN 80-01-02435-0

[13] SAPIETOVÁ, A., DEKÝŠ, V., JAKUBOVIČOVÁ, L., NOVÁK, P., SAPIETA, M. (2023). *Dynamika riešená v programoch Matlab a Msc Adams. (Dynamics Solved in Matlab and Msc Adams)*, EDIS - vydavatelstvo UNIZA, pp. 362, Žilina, 2023, ISBN 978-80-554-1956-5

[14] SHI, L.; SUN, B. (2019). Mathematical modeling and vibration simulation of an orbital sander. *Vibroengineering Procedia 2019*, 22, pp. 176–181. https://doi.org/10.21595/vp.2018.20466

[15] FERNANDES, M.H., GARITAONANDIA, I., ALBIZURI, J., HERNÁNDEZ, J.M., BARRENETXEA, D. (2009). Simulation of an active vibration control system in a centerless grinding machine using a reduced updated FE model. *International Journal of Machine Tools and Manufacture 2009*, 49, no. 3-4, pp. 239-245. ISSN 0890-6955. From: https://doi.org/10.1016/j.ijmachtools.2008.11.002.

[16] LOVSKA, A., GERLICI, J., DIŽO, J., RUKAVISHNIKOV, P. (2026). Study of Strength of Open Wagon with Tank Containers Under Operational Modes. *Eng.* Proc. *2026*, 121, 2. https://doi.org/10.3390/engproc2025121002

[17] ROJÍČEK, J.; PAŠKA, Z.; FUSEK, M.; CIENCIALA, J.; LIČKOVÁ, D. (2022). Finite Element Simulation of HELICOIL® Inserts. *Appl. Sci.* 2022, 12, 11337. https://doi.org/10.3390/app122211337

[18] CERNOHLAVEK, V.; KLIMENDA, F.; HOUSKA, P.; SUSZYŃSKI, M. (2023). Vibration Measurements on a Six-Axis Collaborative Robotic Arm—Part I. *Sensors 2023*, 23, 1629. ISSN: 1424-8220, https://doi.org/10.3390/s23031629

[19] PONIKELSKY, J., CERNOHLAVEK, V., STERBA, J., HOUSKA, P. (2023). Research of Robots in Cooperative Mode in Human Body Part Detection. In: *Manufacturing Technology*. Vol. 23, No. 1 (2023) pp 99-109. ISSN: 1213-2489. DOI: 10.21062/mft.2023.007

[20] MELLER A, SUSZYŃSKI M, LEGUTKO S, TRĄCZYŃSKI M, MRÓZ A, CERNOHLAVEK V. (2023). Optimizing Shock Absorber Operation for Improved Hot Forging Efficiency. *Manufacturing Technology*. 2023;23(5):663-669. doi: 10.21062/mft.2023.074.